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Evaluation of Newtonian Cooling

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Abstract*— *This paper evaluates the accuracy of Newton's law of cooling or Newtonian Cooling at standard conditions at four different temperatures and two different thermal conditions. Graphical data is collected via experiments and other plots are analyzed to quantify the inaccuracy in theoretical models, modeling the observed data. Post-analysis results show that the Law provides higher accuracy in predicting temperatures at lower temperature differences between ambient temperature and the temperature of the object in question; moreover, heat loss through thermal radiation, conduction, and convection increased the accuracy of predicting the temperatures when comparing the accuracy to the case where heat transfer through conduction and radiation was allowed.*

Keywords*— Newton's Law of Cooling· Thermal Radiation· Conduction· Convection· Coefficient of Determination

I. INTRODUCTION

A temperature difference between any two bodies leads to an energy transfer from the hotter to the colder object using typically 3 methods of heat transfer: conduction, convection, and radiation [9]. Modeling the heating and cooling of objects is crucial for designing and optimizing various small-scale and large-scale thermodynamic systems like HVAC control units, heat sinks, and radiators. Several laws and methods exist for modeling the cooling of hot objects with specific strengths and limitations. One such law that incorporates the cooling of hot objects is Newton's law of cooling. Newton's law of cooling first appeared as a verbal definition in 1701, in a short article named 'Scala Graduum Caloris' read by Sir Issac Newton at the Royal Society, which stated that the rate of loss of heat of a hot object is directly proportional to the difference between the object's temperature and the overall temperature of the system or, in other words, the ambient room temperature. Newtonian cooling is one of the strongest theories used to model the temperature change of an object; however, there are some well-researched shortcomings of this law.

Newtonian cooling does not consider the dynamic temperature changes in the surroundings of the thermal system; moreover, the law fails to account for the different media of heat transfer. It works well only where temperature differences are small. More sophisticated attempts at measuring the rate of cooling of a hot object, like the Stefan-Boltzmann law and the Dulong-Petit law utilize different methods of heat transfer in the thermodynamic systems. Stefan-Boltzmann law of cooling can be used to accurately predict the thermal energy transfer of an object and its surroundings if radiation is the only means of energy transfer. The Dulong-Petit law works well for solid substances and is less reliant on assumptions on the system conditions. This law incorporates the thermal properties of solids which makes it more robust than Newtonian cooling. Nevertheless, Newtonian cooling can provide accurate models of the temperature of an object, especially in cases with static constraints like the fluid used and the environment. Evaluative work has been done on Newtonian Cooling using various calorimetric techniques, experimental conditions, and materials. For example, a comparative evaluation by O'Sullivan et al. [11] is performed using Newtonian cooling, Dulong-Petit law, and Stefan-Boltzmann cooling. Vollmer [12] provides a detailed analysis of cooling models using separate laws for conduction and radiation. The study gives insights into the range of temperatures for which Newton's law of cooling gives appreciable results. A few applications are also studied in various research especially relating to nanoparticles and ferrofluids. Researchers have recently proposed new perspectives and refinements to enhance the application of Newton's law of cooling. For instance, Bas et al. [1] introduced a novel approach by considering Newton's law of cooling with fractional conformable derivatives, which yielded improved results compared to traditional integer order derivatives. Additionally, da Silva et al. [4] presented a refined technique for analyzing experimental data using Newton's law of cooling, involving curve fittings and extrapolations to determine temperatures and uncertainties more accurately. Meng et al. [8] proposed a combined device model of a thermoelectric cooler and generator system, if heat transfer obeys Newton's law in the system's operation. Kumar et al. [7] conducted a study on the effect of temperature-dependent properties of fluids on the hydrodynamic and thermal performance of curved tubes under cooling and heating conditions. The study used water and diethylene glycol as different fluids and varied the Reynolds number. Bazgir et al. [2] investigated the effects of performance parameters on thermal separation in a vortex tube with cooling water, showing higher cold air temperature differences and efficiencies compared to a vortex tube without cooling water. Waqas [13] highlighted the use of cooling liquids like water, propylene glycol, and ethylene glycol in various applications, emphasizing the importance of heat transportation enhancement.

While Newton's law of cooling is commonly used in theoretical analyses, some studies have pointed out limitations and deviations associated with its application. For example, Kawashimo et al. [6] noted that the error in Newton's law becomes significant as temperature increases, prompting the exploration of alternative approaches such as Stefan-Boltzmann's law of radiation. Newton's law of cooling remains a valuable tool for analyzing heat transfer phenomena, although researchers continue to explore its limitations and refine its application in various contexts. A brief history of cooling laws is provided by Besson [3] which shows why Newton's law of cooling was and is widely used despite its obvious shortcomings. This paper evaluates the extent to which Newton's law of Cooling models observed data by using hot water as the test fluid. Moreover, the paper looks at the methods to increase the accuracy of the law in modeling the experimental data. The experimental data gathered was used to mathematically derive the modeling equation. The theoretical equations were used to compare the observed cooling curve of water at different temperatures. Initial observations show that the theoretical model is more accurate when all three methods of heat transfer (radiation, conduction, and convection) are allowed in the thermodynamic system; the paper also discusses a different assessment of Newtonian cooling to increase the reliability of the cooling constant k to improve its accuracy.

II. NEWTONIAN COOLING

Newtonian cooling states that the rate at which a body loses heat is proportional to the difference in temperature between the body and the surroundings [5]. The law can be mathematically represented as

$$\frac{dT}{T_f - T_a} = kdt$$

$$\frac{dT}{dt} = k(T_f - T_a)$$

$$\int_{T_i}^{T_f} \frac{dT}{T - T_a} = \int_0^t kdt$$

$$ln(T_f - T_a) - ln(T_i - T_a) = k(t - 0)$$

$$ln\frac{T_f - T_a}{T_i - T_a} = kt$$

$$e^{ln\frac{T_f - T_a}{T_i - T_a}} = e^{kt}$$

$$\frac{T_f - T_a}{T_i - T_a} = e^{kt}$$

$$T_f = T_a + (T_i - T_a)e^{kt.} \qquad (1)$$

Where T_f is the temperature at time t; T_i is the initial temperature; T_a is the ambient temperature; k is the cooling constant.

To derive a usable model for a thermodynamic system, the value of the cooling constant k is generally determined by substituting the surrounding temperature, the initial temperature of the object, and the temperature readings of the object at t = 1; for this paper, the experimental data was collected at a rate of 1 sample per minute. The general form of the cooling constant k for this paper can be written as the following:

$$ln\left(\frac{T_1 - T_a}{T_i - T_a}\right) = k \qquad (2)$$

Where T_1 is the temperature at t = 1; T_i is the initial temperature; T_a is the ambient temperature; k is the cooling constant.

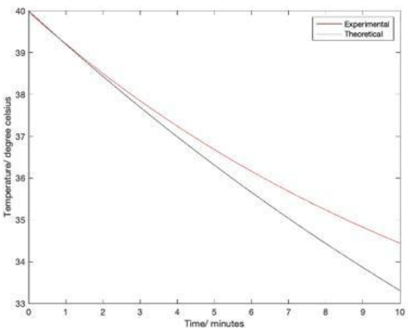
III. METHODOLOGY

To assess the accuracy of the law, two scenarios were synthesized which allowed heat loss through different methods of heat transfer. In the first scenario, 150 mL of boiling tap water at 100° C was added to a 200 ml borosilicate beaker, and the top of the beaker was covered by a thick wooden plank with a tiny hole for the digital thermometer probe, to allow heat loss only through radiation. In the second scenario, the previous setup was replicated but the top was left open to allow heat loss through convection currents and radiation. The setup was placed on a granite platform and it was ensured that no sudden drafts interrupted the cooling of the water beaker. The temperature readings were digitally recorded via a Vernier Go Direct® Temperature Probe. The experimental data was collected at different temperatures of water, specifically at 40° C, 60° C,80° C, and 100° C. Temperature was recorded every minute for a span of 10 minutes for each of the four initial temperature points of water. The experimental data was used to model the equation for each temperature point using the equations (1) and (2). To plot the curve of best fit for the experimental data, MATLAB was used. A statistical measure called 'Coefficient of determination' was used to assess the quality of the curve of best fit. The coefficient of determination (R_2) quantifies the proportion of variance explained by a statistical model and a higher coefficient of determination shows that the model closely predicts the observed data [10]. The equations of the curve of best fit did not follow the format defined by Newton's law in (1) but rather followed a general form of:

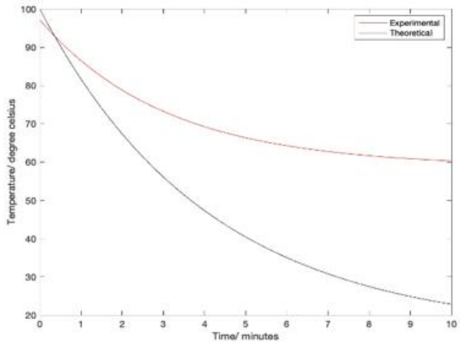
$$ab^t + c \qquad (3)$$

IV. RESULTS

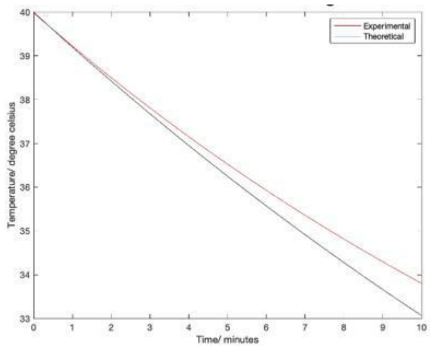
The theoretical equations that model the experimental data took the form presented in (1) and were plotted against the curves of best fit for each temperature. The plots for scenario 1 and scenario 2 for temperatures 40° C and 100° C are given below.



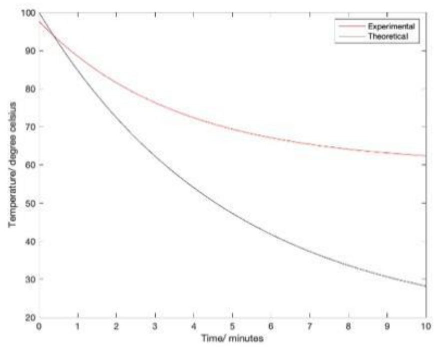
(a) *Initial Temperature: 40°C Scenario 1: Convection attenuated*



(b) *Initial Temperature: 100°C Scenario 1: Convection attenuated*



(a) *Initial Temperature: 40°C Scenario 2: Convection allowed*



(b) *Initial Temperature: 100°C Scenario 2: Convection allowed*

Table 1: Coefficient of determination for Scenario 1, convection attenuated

Initial Temperature/°C	R_2
40	0.9996
60	0.9994
80	0.9991
100	0.9691

Table 2: Coefficient of Determination for Scenario

Initial Temperature/°C	R_2
40	0.9998
60	0.9999
80	0.9999
100	0.9797

2, convection allowed

It can be observed that out of the two scenarios, scenario 2- where convection, along with radiation, was allowed- performed much better in modeling the experimental data as compared to the plots of scenario 1. In the experiment environment above, the average room temperature was around 17.1℃; as the difference between the ambient temperature and the object's initial temperature increases (from 40$^{\circ}C$ to 100$^{\circ}C$), the weaker the model gets as evidenced by an increase in the deviation between experimental and theoretical curves. The y-intercept of Figures 1b and 2b for both the experimental and theoretical plots is not at 100 $^{\circ}C$ as there are some experimental inaccuracies in the measurement. This trend is followed over the range of temperatures mentioned in Tables 1, and 2. As the initial temperature of water increases, the deviation from the curve of best-fit increases. This follows the limitations of Newtonian cooling which states that the law works well with smaller temperature differences.

V. DISCUSSION

A. Observing the error effect of the theoretical model on different values of k

Keeping in mind the significant deviation of the curve produced by Newton's law of cooling from the observed data, the effect of variance in the cooling constant k on this deviation is measured to establish a better representation of the observed data following Newton's law of cooling. Since the data was collected for 10 minutes, values of k are calculated by taking temperature and time readings at time t = 3, 6, and 9 minutes. This is done to investigate which reading provides a better graph with relatively less deviation from the original plot. The graph with the maximum visible deviation, which was scenario 1 with the initial temperature at 100$^{\circ}C$ (Fig. 2), was chosen at first to analyze whether a different approach to calculating k improves the accuracy of Newton's law of cooling.

To determine the value of k, the following equation from the derivation of (1) was taken:

$$k = \frac{ln\left(\frac{T_t - T_a}{T_i - T_a}\right)}{t} \quad (4)$$

where T_t is the temperature of the object at time t; T_a is the ambient temperature; T_i is the initial temperature of the object. k is the cooling constant.

The graph depicting this variation is presented below.

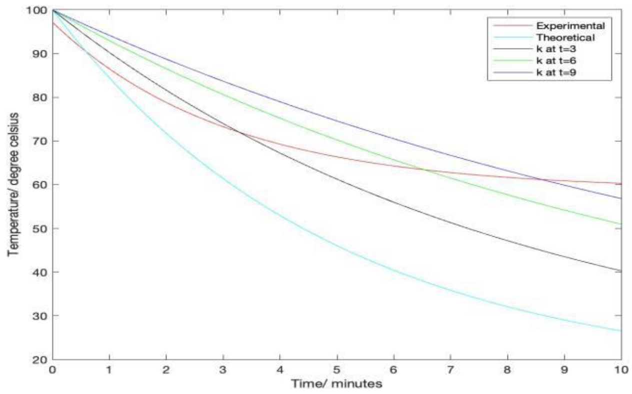


Fig.3: Plots at different values of k

As observed in Fig. 3, the closest plot to the actual data is observed with the equation with the cooling constant value at time t = 6. Additionally, the y-intercept of Fig. 3 for both the experimental and theoretical plots is not at 100 $^{\circ}C$ as there are some experimental inaccuracies in the measurement. This shows that it is possible to get better approximations of observed data with modifications in the method used to calculate the cooling constant in Newtonian cooling.

B. Error Analysis

To quantify the deviation of the theoretical models derived from Newton's law of cooling, the Integral Absolute Error method was used.

Consider two real-valued functions f(x) and g(x) defined for $x|x \in [0,10]$, where f(x) is defined for the experimental model and g(x) is defined for the theoretical model; the formula to calculate the deviation or error percentage of g(x) with respect to f(x) is given as

$$E = \frac{\int_0^{10} |f(x) - g(x)| dx}{\int_0^{10} |f(x)| dx} \times 100$$

The error percentages for the variations are tabulated below.

Table 3: Scenario 1

Initial Temperature/°C	E of model with k at t = 1	E of model with k at t = 3	E of model with k at t = 6	E of model with k at t = 9
40	1.177	0.7878	0.3418	0.4247
60	0.2068	0.2176	0.0927	0.06766
80	3.584	1.854	0.8991	1.181
100	33.01	11.01	7.241	8.825

Table 4: Scenario 2

Initial Temperature/°C	E of model with k at t = 1	E of model with k at t = 3	E of model with k at t = 6	E of model with k at t = 9
40	0.8311	0.349	0.196	0.2249
60	1.0871	0.5672	0.2693	0.3193
80	3.165	1.517	0.844	1.101
100	27.64	9.01	5.734	6.723

VI. CONCLUSION

The data in Tables 3 and 4 corroborate the initial assumption that changing the values of the cooling constant will yield better results. For each case, apart from the anomaly of Table 3 at 60°C, the error percentage is the lowest when the k is calculated with the corresponding values at time t = 6 minutes. As the difference between the temperature of the water and the temperature of the room increases, the error percentage is higher which means that the law is less accurate at higher temperature differences. Furthermore, between the two thermal settings, Newton's law of cooling was more accurate in modeling the observed data in scenario 2 where heat transfer through all mediums was allowed. For this experiment and study, Newtonian cooling produced the best results in modeling experimental data when thermal convection and radiation were allowed and when the cooling constant k was measured using the temperature at the 6th minute in the sample size of 10 minutes. There can be other variations in this experimental setup to increase precision or study different properties of Newtonian cooling. The experiment can also be conducted over different fluids at different sampling rates to further corroborate the results in this paper. Newton's law of cooling is, to date, one of the extensively used theories to simulate and manufacture products and study natural phenomena, and increasing its reliability and accuracy will greatly benefit upcoming products and research.

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