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Computational analysis of the transition of a system between two non-equilibrium stationary states through two-dimensional laminar natural convection in a cylindrical cavity

Herimiah Rakotondranja Stelarijao Eloi[1], Raminosoa Andrianary Lala[2], Ramanantsoa Ravo Mparany[2], Randrianandraina Hery Zo[2], Rakotomalala Minoson[2], Razafinjato Victor Albert[3]

[1]Laboratoire de Thermodynamique Thermique et Combustion, University of Antananarivo
[2]Institut pour la Maitrise de l'Energie d'Antananarivo
[3]Institut Supérieur de Technologie d'Antananarivo

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***Keywords*— Natural convection, non-equilibrium stationary states, transient regime, cylindrical crescent, bi-cylindrical coordinates, laminar regime.**

***Abstract*— *Our work focuses on the numerical study of two-dimensional and transient natural convection in a fluid confined within a crescent-shaped space delimited by two horizontal cylinders. The upper wall is subjected to a non-uniform heat flux, while the lower wall experiences a uniform heat flux, thereby generating thermal natural convection. The transfer equations are solved in a bi-cylindrical coordinate system using the formalism of stream function and vorticity, and then integrated using the finite difference method. Subsequently, these transfer equations are integrated using S.V Patankar's finite difference method with an implicit scheme. The computational program is implemented using Maple V Release Student software. The discretization of the equations highlights the following parameters: the Prandtl number (Pr), the modified Grashof number (Gr), and the aspect ratio (r_2/r_1). The Prandtl number is fixed at 0.7. The results include temperature distributions, local and average Nusselt values, as well as graphs illustrating variations in various parameters based on slice indices.*

I. INTRODUCTION

Natural convection is a heat transfer mechanism that occurs exclusively within fluid mediums when there is a temperature gradient between two surfaces. This mechanism is the most significant mode of heat transfer and involves the description of fluid movement generated by Archimedean forces resulting from variations in density with temperature. Consequently, there is a coupling of dynamics and thermodynamics. The velocity field transports heat and, due to the temperature-dependent density, influences the distribution of mass; in turn, changes in mass create movement through Archimedean buoyancy.

The study of natural convection phenomena captivates researchers due to its widespread applications in various natural phenomena and industrial processes, including the cooling of electronic and electrical components, thermal power plants, nuclear power plants, space heating, heat exchangers, aerospace applications, and even in the vicinity of the human body, among others.

This type of fluid flow is omnipresent in daily life and prevalent in almost all industrial environments. Numerous studies have been conducted on natural convection, focusing on scenarios such as cylinders with walls subjected to uniform density flux [11] or maintained at constant temperatures [4, 22].

Rolland Aimé ANDRIAMAHENINA [8] conducted a study on transient laminar natural convection between two equilibrium states in a fluid confined within a flattened half-ellipsoid, with the wall subjected to a constant density flux.

All the aforementioned studies rely on a mathematical model based on the Boussinesq hypothesis and the two-dimensionality of the flow.

II. MATHEMATICAL MODEL OF TRANSFER EQUATIONS AND NUMERICAL METHOD

Figure 1 illustrates the cross-section of a cylindrical crescent delimited by the intersection of two cylinders, while **Figure 2** depicts the schematic representation of bi-cylindrical coordinates according to [20].

We make the following simplifying assumptions:

- The lower wall is subjected to a uniform heat flux q_2, and the upper wall is traversed by a variable heat flux q [1].

$$q = \frac{\left(I_1 - I\right) N + 1}{I_1} q_2 \tag{1}$$

N: slice index

I_1: total length of the arc where the heat flux q is applied to the upper wall

I: length of the arc where the heat flux q is applied to the upper wall

- The fluid is an ideal gas assumed to be incompressible.
- Viscous dissipation and radiation are considered negligible.
- The physical properties of the fluid are constant, except for its density ρ, which varies and gives rise to natural convection.
- The Boussinesq hypothesis, upon which the heat flux is applied, is valid.
- The convection is laminar and in a transient regime.

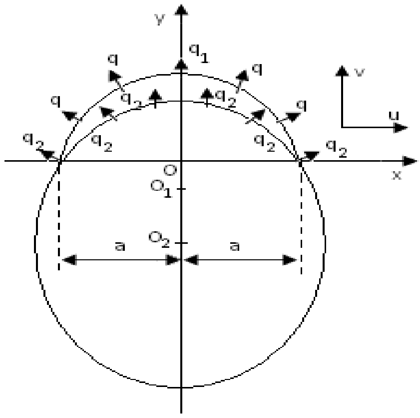

Fig. 1

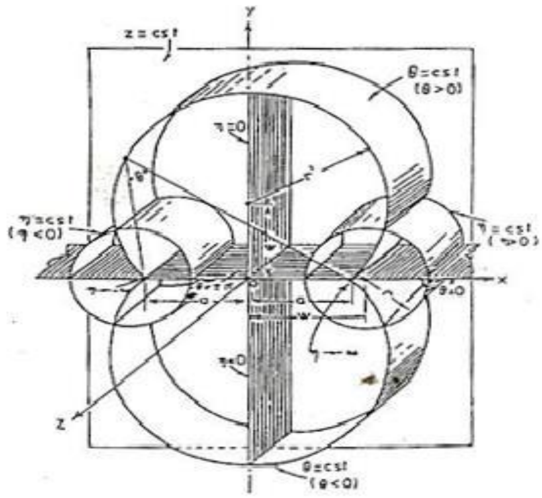

Fig. 2

2.1 Formulation of equations

By introducing vorticity and the stream function, the dimensionless transfer equations in bi-cylindrical coordinates can be expressed as follows:

- Continuity equation

$$\frac{\partial}{\partial \eta}\left(HV_\eta^+\right) + \frac{\partial}{\partial \theta}\left(HV_\theta^+\right) = 0 \tag{2}$$

- Momentum equation

$$\frac{\partial \omega^+}{\partial t^+} + \frac{V_\eta^+}{H}\frac{\partial \omega^+}{\partial \eta} + \frac{V_\theta^+}{H}\frac{\partial \omega^+}{\partial \theta} = \frac{1}{H}\left[F(\eta,\theta)\frac{\partial T^+}{\partial \theta} + G(\eta,\theta)\frac{\partial T^+}{\partial \eta}\right] + \frac{1}{H^2}\left[\frac{\partial^2 \omega^+}{\partial \eta^2} + \frac{\partial^2 \omega^+}{\partial \theta^2}\right] \tag{3}$$

- Heat Equation

$$\frac{\partial T^+}{\partial \eta} + \frac{V_\eta^+}{H}\frac{\partial T^+}{\partial \eta} + \frac{V_\theta^+}{H}\frac{\partial T^+}{\partial \theta} = \frac{1}{\Pr H^2}\left[\frac{\partial^2 T^+}{\partial \eta^2} + \frac{\partial^2 T^+}{\partial \theta^2}\right] \tag{4}$$

2.2 Boundary Conditions

The boundary conditions associated with the transfer equations on both walls are as follows:

- Lower wall (wall with index 2)

  Conditions on velocities and flux:

$$\begin{cases} V_\eta^+\Big|_{(\eta,\theta_2,t^+)} = 0 \\ V_\theta^+\Big|_{(\eta,\theta_2,t^+)} = 0 \end{cases} \quad (5)$$

$$\frac{\partial \psi^+}{\partial \eta}\Big|_{(\eta,\theta_2,t^+)} = 0 \quad (6)$$

$$\frac{\partial T^+}{\partial \theta}\Big|_{(\eta,\theta_2,t^+)} = -q_2\frac{Hg\beta D_H^4}{\lambda v^2} \quad (7)$$

- Upper wall (wall with index 1)

$$\begin{cases} V_\eta^+\Big|_{(\eta,\theta_1,t^+)} = 0 \\ V_\theta^+\Big|_{(\eta,\theta_1,t^+)} = 0 \end{cases} \quad (8)$$

$$\frac{\partial \psi^+}{\partial \eta}\Big|_{(\eta,\theta_1,t^+)} = 0 \quad (9)$$

$$\frac{\partial T^+}{\partial \theta}\Big|_{(\eta,\theta_1,t^+)} = -q\frac{Hg\beta D_H^4}{\lambda v^2} \quad (10)$$

2.3 Numerical method

We solved the system of transfer equations with associated boundary conditions using the "finite difference" method, which relies on TAYLOR series expansions approximating the values of derivatives at a point or in its vicinity through differences. To discretize the equations and boundary conditions, we chose the method developed by S.V. Patankar and Nogotov [6].

III. RESULTS AND DISCUSSION

In our study, we selected air as the fluid, and its physical properties are provided at the initial temperature T_0 = 293 K, corresponding to a Prandtl number Pr = 0.7. The values of physical constants are fixed as follows:

- Focal distance a = 0.12 m.
- Heat flux density q_2 = 12 W/m², resulting in a Grashof number Gr = 10^6.
- All presented results are calculated based on the dimensionless time step of 3.65×10^{-4}.

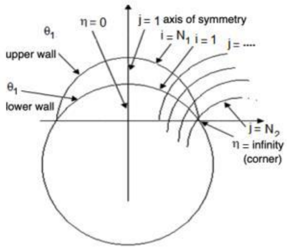

Fig. 3: Notation Representation.

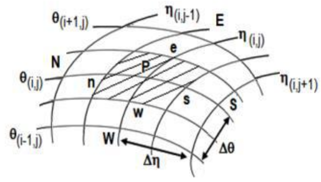

Fig. 4: Node representation in the mesh.

Figure 5 shows the radial variations of dimensionless tangential velocity as a function of the slice index η. Three distinct zones are observed over dimensionless time:

- Near the axis of symmetry (η = 0), the particle velocity magnitude decreases rapidly and approaches zero, as the temperature is very low in this zone (20%);
- In the central zone (between 20% and 80%), the particle velocity magnitude is nearly uniform, following the geometric shape of the crescent, as there is no temperature variation in this range;
- Near the crescent tip, the particle velocity magnitude increases exponentially because the temperature is considerable in this region.

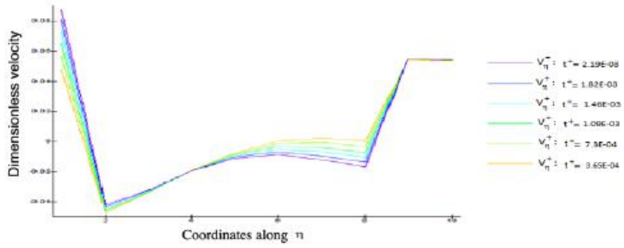

Fig. 5: Radial variations of dimensionless tangential velocity as a function of slice index η over dimensionless time.

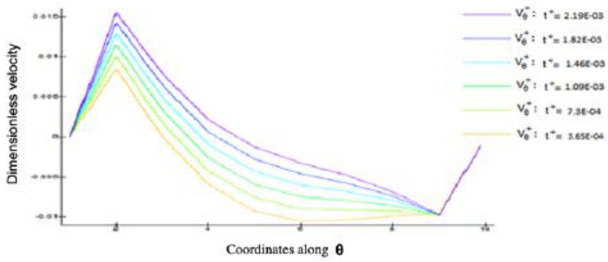

Fig. 6: Radial variations of dimensionless normal velocity as a function of slice index θ over dimensionless time.

Figure 6 depicts the radial variations of dimensionless normal velocity as a function of the slice index θ. The heat flux density through the wall affects the movement of fluid particles, accelerating their velocity, especially when substantial.

The normal velocity curves exhibit alternations:

- Near the axis of symmetry (between 0–20%), an upward movement starting from zero velocity characterizes this range, with a predominance of normal velocity up to the upper part;
- Then a descent (between 20–90%) to the lower part;
- Reaching the lower part, fluid particles ascend, passing through zero normal velocity, and then the cyclic movement of fluid particles recommences.

These phenomena are interpreted by the fact that the movement and normal velocity of fluid particles are influenced by the variation of the variable heat flux density q imposed on the upper wall.

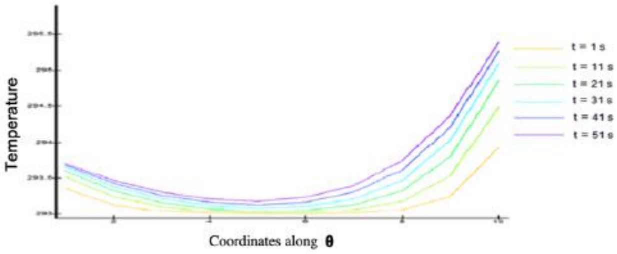

Fig. 7: Variation of real temperature as a function of slice index θ over dimensionless time.

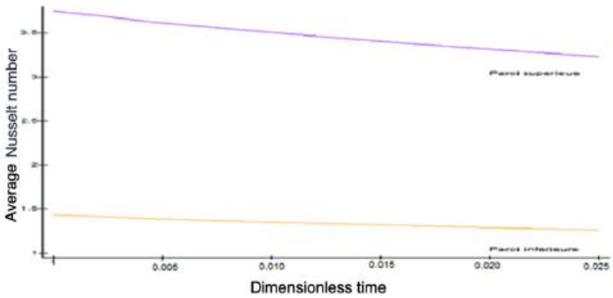

Fig. 8: Variation of the average Nusselt number as a function of dimensionless time.

Figure 7 depicts the variations in real temperature as a function of the slice index θ. It is observed that the temperature decreases slowly and levels off between 30–50%, indicating the existence of a steady-state regime in this zone. Temperatures reach their maximum values at the crescent tip, where temperature variations are substantial.

Figure 8 shows the variations in the average Nusselt number as a function of dimensionless time. The average Nusselt number decreases as dimensionless time increases, indicating a reduction in the temperature gradient, i.e., the fluid temperature begins to approach that of the walls.

IV. CONCLUSION

We conducted a numerical study on the transition of a system between two non-equilibrium stationary states through two-dimensional laminar natural convection in a cylindrical crescent. The use of the finite difference method by S.V. Patankar [5] allows for the approximation of complex partial differential transfer equations to linear partial differential equations. The choice of the bi-cylindrical coordinate system is crucial and suitable for the crescent, given the geometric properties of the system. With the aid of computational tools where the program was executed, we obtained reliable and consistent results regarding radial and tangential velocities, temperature, and the average Nusselt number, in accordance with the adopted methods.

NOMENCLATURE

a: focal distance [m]

Cp: specific heat capacity of the fluid at constant pressure [J.kg.K^{-1}]

D_H: characteristic length scale defined by $D_H = 2(r_2 - r_1)$ [m]

g: acceleration due to gravity [m.s^{-2}]

F, G: functions defined in the momentum equation

h: metric coefficient of the bi-cylindrical coordinate system [m]

H: dimensionless value of h

P: pressure within the fluid [Pa]

q, q_2: respective heat flow densities applied to the upper and lower walls [W.m^{-2}]

q^+: dimensionless heat flux density

r_1, r_2: respective radii of cylinders (C_1) and (C_2) [m]

T: fluid temperature [K]

T^+: dimensionless fluid temperature

T_1: temperature of the lower cylinder [K]

T_2: temperature of the upper cylinder [K]

ΔT: temperature difference defined by $\Delta T = T_2 - T_1$ [K]

T': temperature difference defined by $T' = T - T_1$ [K]

T_r: reference temperature [K]

t: time [s]

t^+: dimensionless time

V_η, V_θ : velocity components in the η and θ directions [m.s^{-1}]

V_η^+, V_θ^+ : dimensionless velocity components in the η and θ directions

$\vec{V}$: velocity vector with components (U, V, W) [m.s⁻¹]

X, Y, Z: Cartesian coordinates [m]

X^+: dimensionless coordinate defined by $X^+ = X/D_H$

α: thermal diffusivity of the fluid $\alpha = \lambda/\rho Cp$ [m².s⁻¹]

β: coefficient of thermal expansion of the fluid at constant pressure, defined by

$$\beta = -\frac{1}{\rho}\left(\frac{\partial P}{\partial T}\right)_P \text{ [K}^{-1}\text{]}$$

λ: thermal conductivity of the fluid [W.m⁻¹.K⁻¹]

η: kinematic viscosity of the fluid [m².s⁻¹]

ρ: density of air [kg.m⁻³]

ω: vorticity [s⁻¹]

ω^+: dimensionless vorticity

η, θ, Z: components in bi-cylindrical coordinates

Ψ: stream function [m².s⁻¹]

Ψ^+: dimensionless stream function

Indices:

Upper Wall

Lower Wall

E, W: East and West nodes, respectively

N, S: North and South nodes, respectively

e, w: East and West faces of the control volume, respectively

n, s: North and South faces of the control volume, respectively

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