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Study on Control of Inverted Pendulum System Based on Simulink Simulation

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***Abstract*— This study aims to conduct control research on an inverted pendulum system using the Simulink simulation platform. The inverted pendulum system is a classic nonlinear dynamic system with important theoretical and practical applications. Firstly, establish a mathematical model of the inverted pendulum system, including the dynamic equation of the pendulum rod and the sensor measurement model. Subsequently, the PID (proportional integral differential) controller design method based on the inverted pendulum system and the fuzzy PID controller design methods were verified through simulation experiments. The ultimate goal is for the designed fuzzy PID controller to effectively stabilize the inverted pendulum system in the vertical position and achieve fast tracking of the target position. Simulation and experimental results show that compared to traditional PID controllers, fuzzy PID controllers can quickly stabilize the pendulum in the target position and have good practicality, stability, speed, and accuracy. Future research can further explore the application of other advanced control strategies in inverted pendulum systems, as well as their potential applications in practical engineering.*

I. INTRODUCTION

The initial research on inverted pendulums began in the 1950s, designed by control theory experts at the Massachusetts Institute of Technology (MIT) in the United States based on the principle of rocket launch boosters. The inverted pendulum system, as the model foundation for shipborne radar, rocket launch systems, and satellite attitude control, has been the focus of many researchers in the past few decades. The research on inverted pendulums will tend towards more complex and in-depth studies. Inverted pendulum systems can be divided into linear inverted pendulums, planar inverted pendulums, composite inverted pendulums, etc. according to their composition. According to their complexity, they can be divided into primary inverted pendulum systems, secondary inverted pendulum systems, tertiary inverted pendulum systems, and multi-level inverted pendulum systems. The first-level inverted pendulum system consists of a driving motor, a conveyor belt, a pendulum rod, a small car, and a test bench [1, 2, 3]. The first-level linear inverted pendulum system is driven by an electric motor and is an unstable, nonlinear, single-input, double-output, strongly coupled system [4, 5, 6, 14, 15]. It controls both the angle of the pendulum and the position of the trolley to be stable, and the steady-state errors of the trolley position and pendulum angle must be controlled within a small range.

The control of the inverted pendulum is a difficult point in the study of inverted pendulum control, and there have been many studies on the inverted pendulum, which are basically based on the assumption that the trolley track of the inverted pendulum system is sufficiently long [7]. With the development of technology, new control methods are constantly emerging, and people use inverted pendulums to test whether new control methods can handle multivariable, nonlinear, and absolute instability. The inverted pendulum has become an ideal experimental method for testing the effectiveness of control strategies [8]. This article focuses on a first-order inverted pendulum system. Firstly, a mathematical model is established using the knowledge of Newtonian mechanics, and then a simulation model of the inverted pendulum system is established using the Simulink module of MTALAB [9, 10, 11, 13]. Finally, by comparing and analyzing the curves and parameters of the established traditional PID controller and fuzzy PID controller, this study is trying to find out whether the fuzzy PID control method is better or not than the ordinary PID control method in terms of stability and speed.

II. ESTABLISHING MATHEMATICAL MODEL

The working principle of a first-order linear inverted pendulum (Figure 1) is that when the data acquisition card transmits the collected data from the rotary encoder to the computer and compares it with the set value. The deviation is processed through some calculation, and a control law is issued to control the motor to make the pendulum swing left and right into the stable range, thereby achieving the pendulum to stand upright and not fall, as well as self-swing [12].

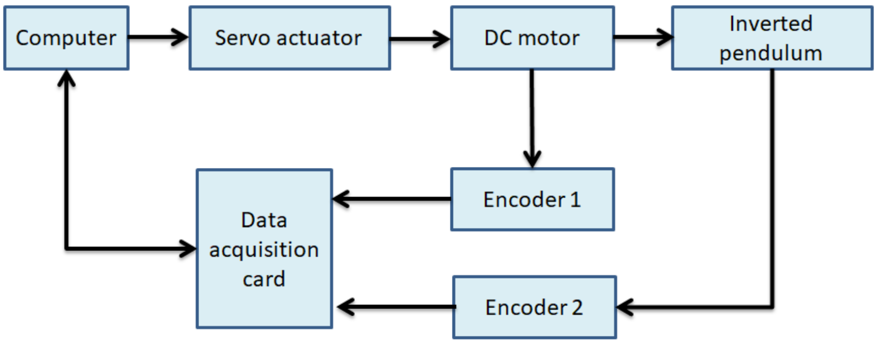

Fig.1 Working Principle Diagram of a First Level Linear Inverted Pendulum

Because establishing a mathematical model of the system is the foundation for studying control methods, the first step is to model the inverted pendulum system in this paper. And mathematical modeling is carried out using the Newtonian mechanics method to obtain the state-space equation of the system and prepare for the subsequent controller design and simulation.

The model parameters of the inverted pendulum system are the pendulum mass m_1=0.109kg, the trolley mass m_2=1.096kg g, the angle between the pendulum and the vertical direction θ (rad), the distance from the center of the swing rod to the car l=0.25m, the distance the car moves x (m), the force applied to the car f (N), the friction coefficient of the car r_f =0.1N/m/sec, the inertia of the swing rod I=0.0034kg*m^2, and the gravitational acceleration g=9.8N/ m^2. The physical model diagram of the inverted pendulum system is shown in Figure 2. A detailed decomposition of various forces acting on the pendulum and trolley using Newtonian mechanics methods is shown in Figure 3. P and N are set as the components of the interaction force in the vertical and horizontal directions during the movement or stability of the car.

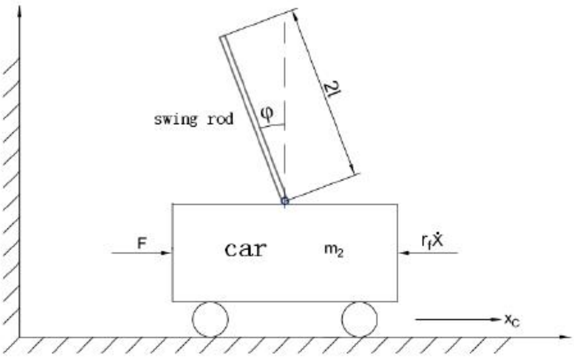

Fig.2 Physical Model of Inverted Pendulum System

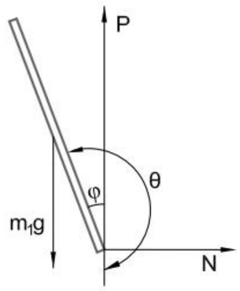

Fig.3 Force Analysis Diagram of the Small Car and Swing Rod

According to Figure 3, first analyze the force in the horizontal direction.

$$m_2\ddot{x} = F - r_f\dot{x} - N \quad (1)$$

$$N = m_1\frac{d^2}{dt^2}(x + l\sin\theta) = m_1\ddot{x} + m_1 l\ddot{\theta}\cos\theta - m_1 l\dot{\theta}^2\sin\theta \quad (2)$$

Substituting equation (2) into equation (1) yields the equation of motion in the horizontal direction

$$(m_1 + m_2)\ddot{x} + r_f\dot{x} + m_1 l\ddot{\theta}\cos\theta - m_1 l\dot{\theta}^2\sin\theta = F \quad (3)$$

In the vertical direction, as the car is used as the horizontal plane, only the force acting on the swing rod needs to be analyzed to obtain

$$P - m_1 g = m_1\frac{d^2}{dt^2}(l\cos\theta) = -m_1 l\ddot{\theta}\sin\theta - m_1 l\ddot{\theta}\sin\theta - m_1 l\dot{\theta}^2\cos\theta \quad (4)$$

The torque balance equation is

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \quad (5)$$

The second equation of motion can be obtained by using equations (2) and (4)

$$(I + m_1 l^2)\ddot{\varphi} - m_1 g l\dot{\varphi} = m_1 l\ddot{x} \quad (6)$$

Set angle θ= π+ φ， and φ after being converted to radians, it is much less than 1 rad, that is $\varphi \ll 1$. So it can be simplified as $\cos\theta = -1$, and $\sin\theta = -\varphi$，$\left(\frac{d\theta}{dt}\right)^2 = 0$.

Using u instead of the input force F, equations (3) and (6) can be simplified as:

$$\begin{cases} (I + m_1 l^2)\ddot{\varphi} - m_1 g l\varphi^{\cdot} = m_1 l\ddot{x} \\ (m_2 + m)\ddot{x} + r_f\,\dot{x} - m_1\,l\ddot{\varphi} = u \end{cases}$$

(7)

Perform a Laplace transform on the above equation, which will be

$$\begin{cases} (I + m_1 l^2)\Phi(s)s^2 - m_1 g l\Phi(s) = m_1 l X(s)s^2 \\ (m_2 + m)X(s)s^2 + r_f X(s)s \; - m_1\,l\Phi(s)s^2 = U(S) \end{cases}$$

(8)

The first equation in the above equation can be written as follows:

$$\frac{\Phi(S)}{X(S)} = \frac{m_1 l s^2}{(I + m_1 l^2) - m_1 g l} \quad (9)$$

Substitute equation (9) into the equation with the control input in equation (8), which will be

$$\frac{\phi(S)}{U(S)} = \frac{\frac{m_1 l}{w}s^2}{s^4 + \frac{r_f(I + m_1 l^2)}{w}s^3 - \frac{(m_1 + m_2)m_1 g l}{w}s^2 - \frac{r_f m_1 g l}{w}s} \quad (10)$$

wherein，$w = [(m_1 + m_2)(I + m_1 l^2) - (m_1 l)^2]$。

Using equation (7) for $\ddot{x}$ and $\ddot{\varphi}$ solve, which will be

$$\begin{cases} \dot{x} = \ddot{x} \\ \ddot{x} = \frac{-(I + m_1 l^2)r_f}{(m_1 + m_2)I + m_1 m_2 l^2}\dot{x} + \frac{m_1{}^2 g l^2}{(m_1 + m_2)I + m_1 m_2 l^2}\varphi + \frac{(I + m_1 l^2)}{(m_1 + m_2)I + m_1 m_2 l^2}u \\ \dot{\varphi} = \ddot{\varphi} \\ \ddot{\varphi} = \frac{m_1 l r_f}{(m_1 + m_2)I + m_1 m_2 l^2}\dot{x} + \frac{m_1 g l(m_1 + m_2)}{(m_1 + m_2)I + m_1 m_2 l^2}\varphi + \frac{m_1 l}{(m_1 + m_2)I + m_1 m_2 l^2}u \end{cases}$$

(11)

Compiled into a standard state space equation as

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I + m_1 l^2)r_f}{(m_1 + m_2)I + m_1 m_2 l^2} & \frac{m_1{}^2 g l^2}{(m_1 + m_2)I + m_1 m_2 l^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_1 l r_f}{(m_1 + m_2)I + m_1 m_2 l^2} & \frac{m_1 g l(m_1 + m_2)}{(m_1 + m_2)I + m_1 m_2 l^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{(I + m_1 l^2)}{(m_1 + m_2)I + m_1 m_2 l^2} \\ 0 \\ \frac{m_1 l}{(m_1 + m_2)I + m_1 m_2 l^2} \end{bmatrix} u \quad (12)$$

$$y = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix}$$

If the pendulum is regarded as a rod with uniform mass, then the inertia of the pendulum is

$$I = \frac{1}{3} m_1 l^2 \tag{13}$$

Substituting equation (13) into the first equation of equation (7) yields

$$\left(\frac{1}{3} m_1 l^2 + m_1 l^2\right) \ddot{\varphi} - m_1 g l \dot{\varphi} = m_1 l \ddot{x} \tag{14}$$

Simplify the above equation, which will be

$$\ddot{\varphi} = \frac{3g}{4l} \varphi + \frac{3}{4l} \ddot{x} \tag{15}$$

Let the system state space equation be

$$\begin{cases} \dot{X} = AX + Bu \\ Y = CX + DU \end{cases} \tag{16}$$

Let $X = [\dot{x} \quad \ddot{x} \quad \dot{\theta} \quad \ddot{\theta}], u' = \ddot{x}$, The following state space expression can be obtained

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\varphi} \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{3g}{4l} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{3}{4l} \end{bmatrix} u' \tag{17}$$

$$Y = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix}$$

III. CONTROLLER DESIGN AND FUZZY LOGIC ESTABLISHMENT

Firstly, traditional PID is used to design PID controllers for the output displacement and output angle of the system. Through parameter tuning and control, the spatial-state equation output reaches a stable state. When studying the displacement of a small car, the spatial state control equation of the system is obtained by inputting the car parameters into equation (17) as follows:

$$\begin{cases} X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 29.4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0.75 \end{bmatrix} u' \\ Y = [x] = [1 \quad 0 \quad 0 \quad 0] X \end{cases} \tag{18}$$

We first use the traditional PID control method to control the spatial-state equation of displacement output and simulate it using Simulink in Matlab. We then adjust a suitable set of PID parameters through parameter tuning principles. The input is the unit step signal, and the PID parameter is K_p= 70, K_I = 0.7, K_D = 6 the system reaches steady state at time T=0.08S.

The fuzzy controller designed here is a two-input, three-output fuzzy PID controller, and the two output signals jointly use a fuzzy rule. The input is the error $e= r(k)-y(k)$ and error change rate $ec = e(k)-e(k-1)$ between the given value and the actual value of the car displacement or swing rod angle, and the output is the corrected values ΔK_p, ΔK_I, and ΔK_D of the PID parameters.

In this design, the basic domain of error e is taken as [-5, 5], and the basic domain of error change rate ec is taken as [-5, 5]. The domain of the output variables ΔK_p, ΔK_I, and ΔK_D are taken as [-3, 3].

In order to obtain the input of the fuzzy controller, it is necessary to fuzzify the precise quantity, that is, multiply the input quantity by the corresponding quantization factor, and convert it from the basic domain to the corresponding fuzzy domain. The quantization factor of error e is α_e=0.8, and the factor of error change rate ec is α_{ec}=0.2. The control quantity obtained through the fuzzy control algorithm is a fuzzy quantity that needs to be multiplied by a proportional factor and converted into the basic domain. When the output variable is the displacement of the car, the scaling factor of $\Delta K_p, \Delta K_I, \Delta K_D$ are **$\alpha_{\Delta Kp} = \alpha_{\Delta KI} = 1, \alpha_{\Delta KD} = -5$.** When taking the output variable swing angle, the scaling factor of $\Delta K_p, \Delta K_I, \Delta K_D$ are **$\alpha_{\Delta Kp}=200, \alpha_{\Delta KI}=1, \alpha_{\Delta KD}=30$**

Divide the fuzzy domain of input variables(e, ec) and output variables ($\Delta K_p, \Delta K_I, \Delta K_D$) into 7 fuzzy subsets, namely NB, NM, NS, ZO, PS, PM, PB representing negative big, negative medium, negative small, zero, positive small, positive medium, and positive big, respectively. The membership functions of input variables and output variables both adopt triangular membership functions, as shown in Figure 4.

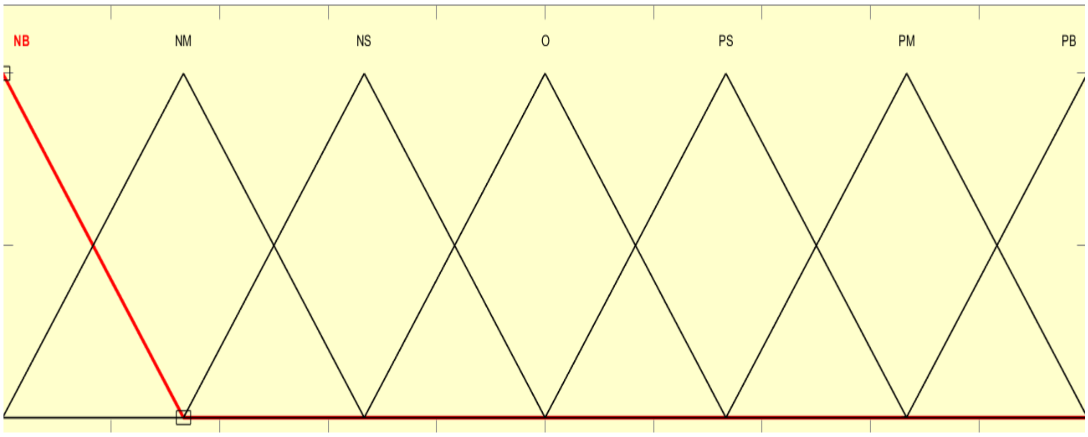

Fig.4 The Membership Function of e, ec,ΔK_p,ΔK_I,ΔK_D

The fuzzy rule statement used by the output variable fuzzy controller is as follows:
"**if *E* is*α*and *EC* is*β*then *U* is*γ***" Wherein,***α*,*β*,*γ***both represent the fuzzy sets corresponding to each variable.

Based on the impact of PID parameters on system performance, parameter tuning principles, expert experience, and cognition, 49 control rules were obtained after processing, as shown in Tables 1–3.

Table 1 Fuzzy Rule Table ofΔK_p

K_P E / EC	NB	NM	NS	O	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	O
NM	PB	PB	PB	PB	PM	O	O
NS	PM	PM	PM	PM	O	PS	PS
O	PM	PM	PS	O	NS	NS	NM
PS	PS	PS	O	NS	NM	NM	NM
PM	PS	O	NS	NM	NM	NM	NB
PB	O	O	NM	NM	NM	NB	NB

Table 2 Fuzzy Rule Table ofΔK_I

K_I E / EC	NB	NM	NS	O	PS	PM	PB
NB	NB	NB	NM	NM	NS	O	O
NM	NB	NB	NM	NS	NS	O	O
NS	NB	NM	NS	NS	O	PS	PS
O	NM	NM	NS	O	PS	PM	PM
PS	NM	NS	O	PS	PS	PM	PB
PM	O	O	PS	NM	PM	PB	PB
PB	O	O	PS	PM	PM	PB	PB

Table 3 Fuzzy Rule Table of ΔK_D

K_D \ E / EC	NB	NM	NS	O	PS	PM	PB
NB	PS	NS	NB	NB	NB	NM	PS
NM	PS	NS	NB	NM	NM	NS	O
NS	O	NS	NM	NM	NM	NS	O
O	O	NS	NS	NS	NS	NS	O
PS	O	O	O	O	O	O	O
PM	PB	PS	PS	PS	PS	PS	PB
PB	PB	PM	PM	PM	PS	PS	PB

This design system uses the Mamdani inference method to perform fuzzy inference on the established fuzzy rules in order to obtain control variables. Meanwhile, using the center of gravity method to solve the fuzziness of language expression, thus obtaining the exact value of $\Delta K_p, \Delta K_I, \Delta K_D$. In addition, the values obtained through fuzzy reasoning and deblurring are multiplied by the corresponding scaling factors to obtain the incremental adjustment values of PID parameters, which are then substituted into equations (19) - (21) to obtain the control parameters of the PID controller.

$$K_P = K_{P0} + \triangle K_P \quad (19)$$

$$K_I = K_{I0} + \triangle K_I \quad (20)$$

$$K_D = K_{D0} + \triangle K_D \quad (21)$$

Before establishing a fuzzy logic controller, the Tool Box parameters of the fuzzy logic system need to be set in Simulink (Figure 5). Then use the membership function parameter setting process of Tool Box (Figure 6) to establish the membership function of *e, ec*, $\Delta K_p, \Delta K_I, \Delta K_D$.

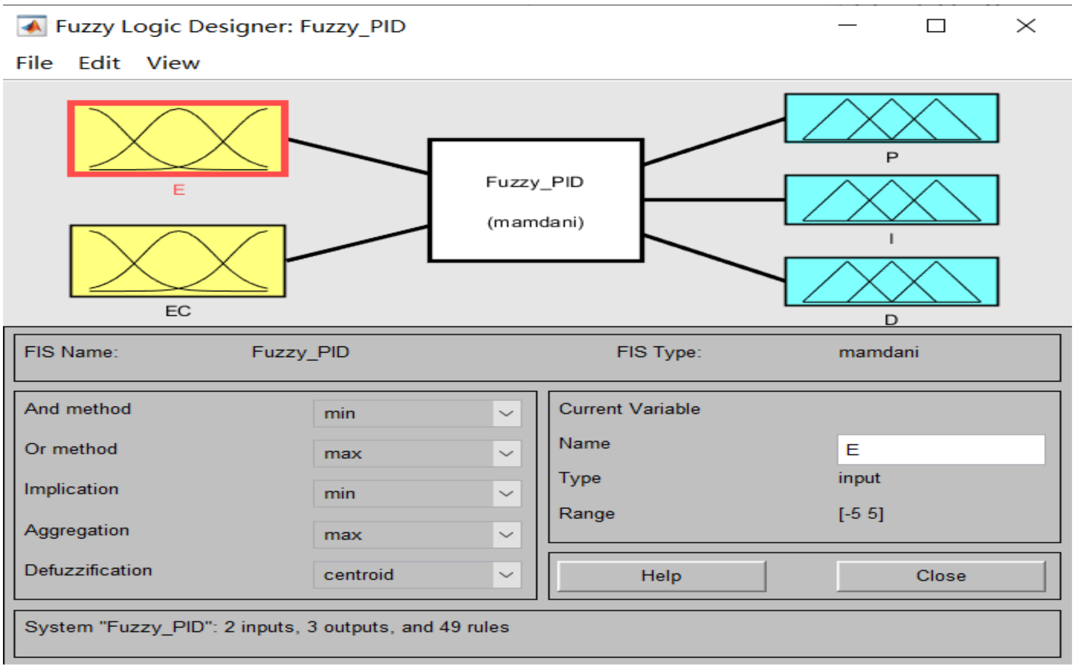

Fig.5 Fuzzy Logic System Tool Box

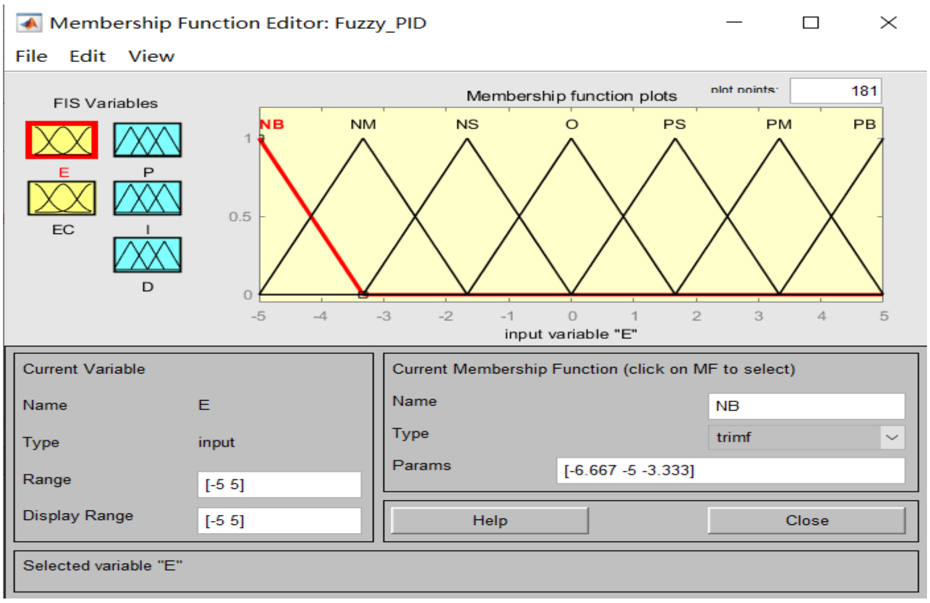

Fig.6 Membership Function of Fuzzy Logic System

After setting the above parameters, they can be added to the rule editor, and all output surfaces of the fuzzy inference system can be observed in the output surface observer.

IV. COMPARISON BETWEEN TRADITIONAL PID CONTROL AND FUZZY PID CONTROL

Through Simulink simulation (Figure 7 and Figure 8), it was found that, under appropriate parameters, traditional PID controllers have good control effects on the control object that can assume an accurate mathematical model. They can meet the requirements of system control accuracy and rise time, but there are problems such as long adjustment times. The fuzzy PID controller can adjust the PID parameters of the system in real time. By doing so, the PID parameters can be more suitable for the control requirements of the system, resulting in better control effects than traditional PID controllers (Figure 9).

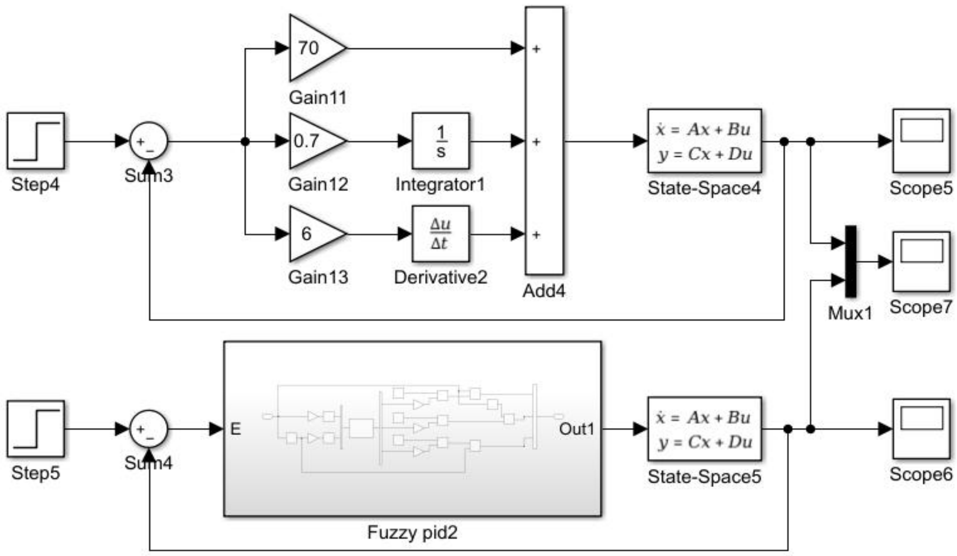

Fig.7 Simulink Simulations of Traditional PID and Fuzzy PID Control Systems (Swing Angle)

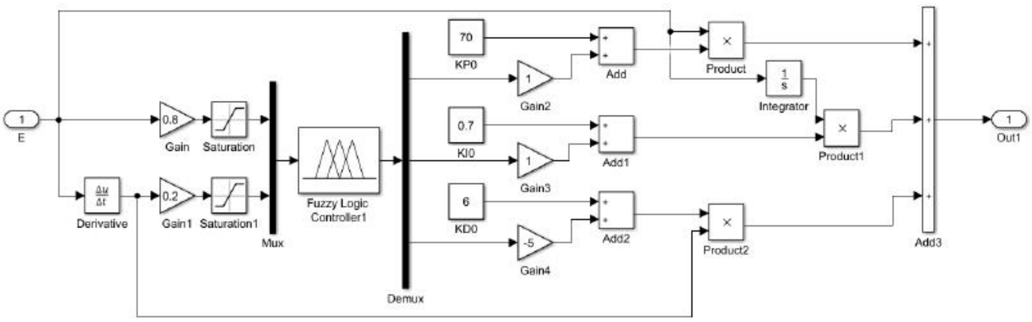

Fig.8 Simulink Simulation of a Fuzzy PID Controller for Swing Angle

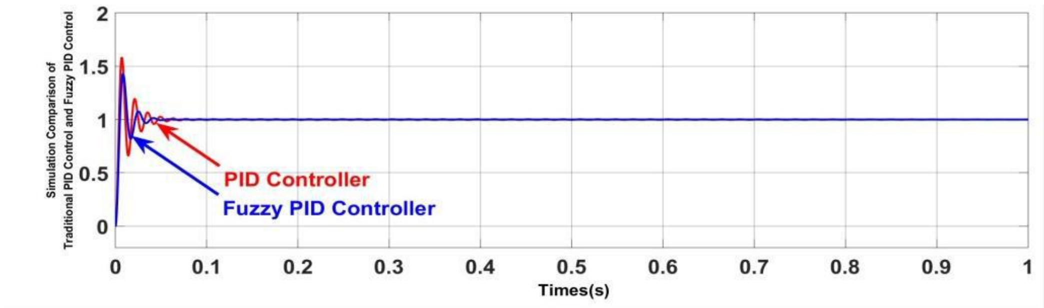

Fig.9 Response Curves of Traditional PID and Fuzzy PID Controller Systems (Swing Angle)

V. CONCLUSIONS

The inherent first-order linear inverted pendulum system is a typical single-input, multiple-output nonlinear system. This article analyzes the motion of the inherent first-order linear inverted pendulum system, establishes a mathematical model of the first-order linear inverted pendulum system using the Newtonian mechanics method, and designs PID controllers and fuzzy PID controllers to control the first-order linear inverted pendulum system separately.

By adjusting their proportional constant and integral constant, the three parameters of the differential constant enable the first-order linear inverted pendulum system to quickly and accurately reach a stable state, ultimately achieving a stable system. The superiority of fuzzy PID controllers over PID control was verified through Simulink simulation. Under the action of a fuzzy PID controller, the system has fast response speed, short adjustment time, and high steady-state accuracy, achieving the expected goals.

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