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Equations of a Synchronous Machine in Phase Coordinates for Asymmetrical Short Circuits and their Solutions

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Abstract*— A methodology is proposed for studying, using digital technology, asymmetrical transient modes of a synchronous generator operating on infinite power systems. The initial equations of a synchronous machine and sequential transformations for their solution are presented. Differential equations of synchronous machines are compiled in phase coordinates. When solving the equations, the parameters of the electrical network included between the generator and the system are taken into account. The results of calculations using the obtained equations for the 14-node IEEE test circuit are presented, confirming the effectiveness of this approach.

I. INTRODUCTION

The analysis of asymmetrical transient processes of synchronous machines becomes of great importance, especially for highly used powerful turbo and hydro generators. As a rule, modern heavy-duty generators operate in fairly large power systems and therefore it is necessary to analyze asymmetrical processes not of single-phase generators (to which a large number of works are devoted and this issue is sufficiently covered in the press), but the operation of generators that lose connection with the power system (for example, with single-phase or two-phase short circuits, with phase-by-phase automatic restarts, etc.) (Basmanov & Votinsev, 2021) (Biswas, Pal, Werho, & Vittal, 2021) (Dong, Tian, & Ding, 2021). Additional difficulties for the analysis are presented by taking into account the reactivity connected between the generator and the system (the reactivity of transformers and power transmission lines (PTL)). Differential equations describing transient asymmetric processes of a synchronous machine contain periodic coefficients and therefore their complete analytical solution using known tabulated functions is practically impossible.

An analytical study of asymmetrical transient processes of a synchronous machine operating on an infinite power network through reactance was carried out quite fully only in (Goldberg, Bul, Sviridenko, & Helemskaya, 2001). The author provides analytical expressions for the stator current only for a two-phase short circuit of the generator and a single-phase short circuit to the neutral of the system and the generator.

The difficulties of solving a complete system of differential equations with periodic coefficients are overcome by numerically solving the problem using computer technology (CT). This work is devoted to the creation of a technique for numerical analysis of some transient asymmetric modes of a synchronous generator operating on an infinite power network using CT (Hugo, Gloria, Alvaro, Jesus, & Nikolas, 2022) (Jaramillo Serna

& López-Lezama, 2019) (Krishna, Sasikala, & Ganesh, 2017) (Pai, 2014).

The article presents the basic equations for calculating transient asymmetric modes on a CT, information on their programming, as well as the results of calculations on a CT.

II. BASIC EQUATIONS FOR CALCULATING ASYMMETRICAL MODES OF A SYNCHRONOUS GENERATOR

The initial equations for the stator winding are compiled in the system of axes a, b, c; for rotor quantities, the system of axes d, q, 0 is used. Periodic coefficients in the equations of a synchronous machine are calculated at each interval of the numerical solution of the equations as a function of the angle between the stator and rotor axes. In this case, its periodic coefficients are expressed only in terms of and calculating them on a CT is not difficult. (Dong, Tian, & Ding, 2021) (Grigsby, 2001) (Guo, Bao, Xiao, & Chen, 2021) (Guseinov & Ibrahimov, 2012)

The advantage of the used axis system over any other is that all current values correspond to real values and do not require recalculation of the results to obtain phase values.

When drawing up equations for calculating asymmetrical transient modes of a synchronous machine, the following assumptions were made:

1. The phase windings of a synchronous machine are symmetrical, i.e. they have the same number of turns, active resistance and mutual shift of magnetic axes.

2. When considering the magnetic fields of the self-induction of the stator windings and the mutual induction of these windings with the rotor windings, only one harmonic of this distribution is taken into account.

3. The magnetic permeability of the machine's magnetic core steel is assumed to be constant. Saturation is taken into account by choosing constant saturated parameter values.

4. It is accepted that on the rotor, in addition to the excitation winding, there is one damper circuit along the longitudinal and transverse axes.

The initial equations for calculating the modes of a synchronous machine in coordinates a, b, c are the following differential equations for the stator winding voltages:

$$\left.\begin{aligned} p\psi_a &= e_a - i_a r_a \\ p\psi_b &= e_b - i_b r_b \\ p\psi_c &= e_c - i_c r_c \end{aligned}\right\}, \quad (1)$$

where ψ_a, ψ_b, ψ_c – flux linkage of stator winding phases; i_a, i_b, i_c – stator winding phase currents; r_a, r_b, r_c – active phase resistance of the stator winding; e_a, e_b, e_c – voltage at the terminals of the generator stator phase windings; $p = \frac{d}{d\tau}$ – differentiation operator with respect to synchronous time $\tau = 2\pi ft$.

To this system of equations one should add the stress equations for the rotor circuits and the rotor motion equations:

$$\left.\begin{aligned} p\psi_f &= e_f - i_f r_f \\ p\psi_{kd} &= -i_{kd} r_{kd} \\ p\psi_{kq} &= -i_{kq} r_{kq} \\ pS &= \frac{1}{H}\left(M_m + M_e\right) \\ p\theta &= S \end{aligned}\right\}, (2)$$

where $\left.\begin{matrix} \psi_f, \psi_{kd}, \psi_{kq} \\ i_f, i_{kd}, i_{kq} \\ r_f, r_{kd}, r_{kq} \end{matrix}\right\}$ – current flux linkage and active resistance of the excitation winding and damper circuits along the longitudinal and transverse axes; e_f – voltage applied to the excitation winding; S – slip; H – inertial constant in electric rads; M_m – torque of the load on the shaft of a synchronous machine; M_e – electromagnetic torque of synchronous machine; θ – working angle (the angle between the transverse axis of the rotor and the representing vector of phase voltages).

To solve systems of equations (1) and (2) using any of the well-known numerical methods of Runge–Kutta, Adams Euler, etc. (Zakaryukin & Kryukov, Complex asymmetric modes of electrical systems, 2005), it is necessary that the number of variables equals the number of equations. Experience shows that it is expedient to express all currents through the flux linkages of the circuits. For this purpose, known relations are used, obtained in the calculations of symmetrical modes using the Park-Gorev equations (Goldberg, Bul, Sviridenko, & Helemskaya, 2001) (Lupkin, 1985) (Kryuchkov & others, 2009) (Kundur) (Yusifbeyli, 2019).

$$\left.\begin{aligned} i_d &= a\psi_d - b\psi_f - c\psi_{kd} \\ i_q &= g\psi_q - h\psi_{kq} \\ i_f &= -b\psi_d + d\psi_f - e\psi_{kd} \\ i_{kd} &= -c\psi_d - e\psi_f + f\psi_{kd} \\ i_{kq} &= -h\psi_q + k\psi_{kq} \\ i_0 &= \frac{\psi_0}{x_0} \end{aligned}\right\}, \quad (3)$$

where coefficients a,b,c,d,e,f,g,h,k are expressed through machine parameters as follows :

$$\left.\begin{aligned} a &= \frac{X_f X_{kd} - x_{ad}^2}{\Delta}; d = \frac{X_d X_{kd} - x_{ad}^2}{\Delta}; g = \frac{X_{kq}}{X_q X_{kq} - x_{aq}^2}; \\ b &= \frac{x_{ad} X_{kd} - x_{ad}^2}{\Delta}; e = \frac{X_d x_{ad} - x_{ad}^2}{\Delta}; h = \frac{x_{aq}}{X_q X_{kq} - x_{aq}^2}; \\ c &= \frac{x_{ad} X_f - x_{ad}^2}{\Delta}; f = \frac{X_d X_f - x_{ad}^2}{\Delta}; k = \frac{X_q}{X_q X_{kq} - x_{aq}^2}; \end{aligned}\right\} \quad (4)$$

$$\Delta = X_d\left(X_f X_{kd} - x_{ad}^2\right) - x_{ad}\left(x_{ad} X_{kd} - x_{ad}^2\right) - - x_{ad}\left(x_{ad} X_f - x_{ad}^2\right).$$

The parameters included in these expressions represent the mutual or total reactivity of the circuits:

$$X_f = x_{ad} + x_f; \quad X_{kd} = x_{ad} + x_{kd}; \quad X_{kq} = x_{aq} + x_{kq};$$
$$X_d = x_{ad} + x_e; \quad X_q = x_{aq} + x_l;$$

To transition from stator currents i_d, i_q, i_0 to phase quantities i_a, i_b, i_c, we use the known relationships equations (Goldberg, Bul, Sviridenko, & Helemskaya, 2001) (Lupkin, 1985) (Prabha & Lei) (Soldatov & Popov, 2004) (Soldatov & Popov, 2005) (Zakaryukin, Kryukov, & Le, 2013).

$$\left.\begin{aligned} i_a &= i_0 + i_d\cos\gamma - i_q\sin\gamma \\ i_b &= i_0 + i_d\cos(\gamma-\rho) - i_q\sin(\gamma-\rho) \\ i_c &= i_0 + i_d\cos(\gamma+\rho) - i_q\sin(\gamma+\rho) \end{aligned}\right\}, \quad (5)$$

$$\left.\begin{aligned} \psi_0 &= \frac{1}{3}(\psi_a + \psi_b + \psi_c) \\ \psi_d &= \frac{2}{3}[\psi_a\cos\gamma + \psi_b\cos(\gamma-\rho) + \psi_c\cos(\gamma+\rho)] \\ \psi_q &= \frac{2}{3}[\psi_a\sin\gamma + \psi_b\sin(\gamma-\rho) + \psi_c\sin(\gamma+\rho)] \end{aligned}\right\}, (6)$$

where $\rho = \frac{2\pi}{3}$ =120° for a machine with symmetrically arranged three phase windings, $\gamma = \tau + \theta + \frac{\pi}{2}$ – angle between the fixed phase axis a and the rotating longitudinal axis of the rotor.

After the transformation, we obtain expressions for currents i_a, i_b, i_c through flux linkages and trigonometric functions of angle γ :

$$\begin{aligned} i_a &= \frac{1}{3x_0}(\psi_a + \psi_b + \psi_c) + \psi_a\left[\frac{2}{3}a - \frac{2}{3}(a-g)\sin^2\gamma\right] + \\ &+ \psi_b\left[-\frac{1}{3}a + \frac{1}{3}(a-g)\sin^2\gamma + \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] + \\ &+ \psi_c\left[-\frac{1}{3}a + \frac{1}{3}(a-g)\sin^2\gamma - \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] - \\ &- \psi_f b\cdot\cos\gamma - \psi_{kd}c\cdot\cos\gamma + \psi_{kq}h\cdot\sin\gamma \end{aligned} \quad (7)$$

$$\begin{aligned} i_b &= \frac{1}{3x_0}(\psi_a + \psi_b + \psi_c) + \psi_a\left[-\frac{1}{3}a + \frac{1}{3}(a-g)\sin^2\gamma + \right. \\ &\left. + \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] + \psi_b\left[\frac{2}{3}a - \frac{1}{2}(a-g) + \right. \\ &\left. + \frac{1}{3}(a-g)\sin^2\gamma - \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] + \\ &+ \psi_c\left[-\frac{1}{3}a + \frac{1}{2}(a-g) - \frac{2}{3}(a-g)\sin^2\gamma\right] - \\ &- \psi_f b\left(\frac{\sqrt{3}}{2}\sin\gamma - \frac{1}{2}\cos\gamma\right) - \psi_{kd}c\left(\frac{\sqrt{3}}{2}\sin\gamma - \frac{1}{2}\cos\gamma\right) - \\ &- \psi_{kq}h\left(\frac{1}{2}\sin\gamma + \frac{\sqrt{3}}{2}\cos\gamma\right) \end{aligned} \quad (8)$$

$$\begin{aligned} i_c &= \frac{1}{3x_0}(\psi_a + \psi_b + \psi_c) + \psi_a\left[-\frac{1}{3}a + \frac{1}{3}(a-g)\sin^2\gamma - \right. \\ &\left. - \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] + \psi_b\left[-\frac{1}{3}a + \frac{1}{2}(a-g) - \right. \\ &\left. - \frac{2}{3}(a-g)\sin^2\gamma\right] + \psi_c\left[\frac{2}{3}a - \frac{1}{2}(a-g) + \frac{1}{3}(a-g)\sin^2\gamma + \right. \\ &\left. + \frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right] + \psi_f b\cdot\left(\frac{1}{2}\cos\gamma + \frac{\sqrt{3}}{2}\sin\gamma\right) + \\ &+ \psi_{kd}c\cdot\left(\frac{1}{2}\cos\gamma + \frac{\sqrt{3}}{2}\sin\gamma\right) - \psi_{kq}h\left(\frac{1}{2}\sin\gamma - \frac{\sqrt{3}}{2}\cos\gamma\right) \end{aligned} \quad (9)$$

$$\begin{aligned} i_f &= -\frac{2}{3}\psi_a b\cdot\cos\gamma + \psi_b b\left(\frac{1}{3}\cos\gamma - \frac{\sqrt{3}}{3}\sin\gamma\right) + \\ &+ \psi_c b\left(\frac{1}{3}\cos\gamma + \frac{\sqrt{3}}{3}\sin\gamma\right) + d\psi_f - e\psi_{kd} \end{aligned} \quad (10)$$

$$\begin{aligned} i_{kd} &= -\frac{2}{3}\psi_a c\cdot\cos\gamma - \psi_b c\cdot\left(\frac{1}{3}\cos\gamma - \frac{\sqrt{3}}{3}\sin\gamma\right) + \\ &+ \psi_c c\cdot\left(\frac{1}{3}\cos\gamma + \frac{\sqrt{3}}{3}\sin\gamma\right) - e\psi_f + f\psi_{kd} \end{aligned} \quad (11)$$

$$i_{kq}=\frac{2}{3}\psi_a h\cdot\sin\gamma+\psi_b h\left(\frac{1}{3}\sin\gamma+\frac{\sqrt{3}}{3}\cos\gamma\right)+$$
$$+\psi_c h\left(\frac{1}{3}\sin\gamma-\frac{\sqrt{3}}{3}\cos\gamma\right)+k\psi_{kd} \quad (12)$$

The magnitude of the electromagnetic torque in the a, b, c axes is obtained after substitution into the well-known formula

$$M_e=\psi_q i_d-\psi_d i_q$$

currents and flux linkages i_d, i_q, ψ_d, ψ_q expressed through $i_a, i_b, i_c, \psi_a, \psi_b, \psi_c$ in the following form:

$$M_e=\frac{2\sqrt{3}}{9}\left[\psi_a(i_c-i_b)+\psi_b(i_a-i_c)+\psi_c(i_b-i_a)\right] \quad (13)$$

Substituting the expressions for currents i_a, i_b, i_c through flux linkages ψ_a, ψ_b, ψ_c, we obtain the final expression for the electromagnetic torque used in calculating transient modes at the CT:

$$M_e=\frac{2\sqrt{3}}{9}(\psi_b-\psi_c)\left\{\frac{1}{3x_0}(\psi_a+\psi_b+\psi_c)+\right.$$
$$+\psi_a\left[\frac{2}{3}a-\frac{2}{3}(a-g)\sin^2\gamma\right]+$$
$$+\psi_b\left[-\frac{1}{3}a+\frac{1}{3}(a-g)\sin^2\gamma+\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]+$$
$$+\psi_c\left[-\frac{1}{3}a+\frac{1}{3}(a-g)\sin^2\gamma-\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]-$$
$$\left.-\psi_f b\cdot\cos\gamma-\psi_{kd}c\cdot\cos\gamma+\psi_{kq}h\cdot\sin\gamma\right\}+$$
$$+\frac{2\sqrt{3}}{9}(\psi_c-\psi_a)\left\{\frac{1}{3x_0}(\psi_a+\psi_b+\psi_c)+\right.$$
$$+\psi_a\left[-\frac{1}{3}a+\frac{1}{3}(a-g)\sin^2\gamma+\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]+$$
$$+\psi_b\left[\frac{2}{3}a-\frac{1}{2}(a-g)+\frac{1}{3}(a-g)\sin^2\gamma-\right.$$
$$\left.-\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]+\psi_c\left[-\frac{1}{3}a+\frac{1}{2}(a-g)-\right.$$
$$\left.-\frac{2}{3}(a-g)\sin^2\gamma\right]-(\psi_f b+\psi_{kd}c)\left(\frac{\sqrt{3}}{2}\sin\gamma-\frac{1}{2}\cos\gamma\right)-$$
$$\left.-\psi_{kq}h\left(\frac{1}{2}\sin\gamma+\frac{\sqrt{3}}{2}\cos\gamma\right)\right\}+$$
$$+\frac{2\sqrt{3}}{9}(\psi_c-\psi_a)\left\{\frac{1}{3x_0}(\psi_a+\psi_b+\psi_c)+\right.$$
$$\psi_a\left[-\frac{1}{3}a+\frac{1}{3}(a-g)\sin^2\gamma-\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]+$$
$$+\psi_b\left[-\frac{1}{3}a+\frac{1}{2}(a-g)-\frac{2}{3}(a-g)\sin^2\gamma\right]+$$
$$+\psi_c\left[\frac{2}{3}a-\frac{1}{2}(a-g)+\frac{1}{3}(a-g)\sin^2\gamma+\right.$$
$$\left.+\frac{\sqrt{3}}{3}(a-g)\cos\gamma\cdot\sin\gamma\right]+(\psi_f b+\psi_{kd}c)\left(\frac{1}{2}\cos\gamma+\right.$$
$$\left.\left.+\frac{\sqrt{3}}{2}\sin\gamma\right)-\psi_{kq}h\left(\frac{1}{2}\sin\gamma-\frac{\sqrt{3}}{2}\cos\gamma\right)\right\}.$$

III. TAKING INTO ACCOUNT THE EXTERNAL REACTIVITY OF THE SYSTEM

The system of equations (1) is compiled for the operation of a synchronous machine in parallel with an infinite power system. In this case, e_a, e_b, e_c - are the voltages on the tires of infinite power.

The reactance and active resistances of transformers and lines from a synchronous machine to infinite power buses are included in the leakage reactance of the stator winding and in the active resistances of phase windings. If an accident occurs somewhere in the middle of a power line, then the voltage at the place of the accident will differ from the voltage of the buses of infinite power. In this case, part of the reactivity of the line from the generator to the point of the short circuit is added to the leakage reactivity and the machine is connected to the network through additional resistance Z_A, Z_B, Z_C, which represents the resistance of the remaining part of the line from the point of the short circuit to the infinite power buses with voltages e_A, e_B, e_C that do not depend on the mode. Let us consider a special case when $z(p)=r+xp$. The system of voltages e_a, e_b, e_c applied to the phases of the stator winding is determined from the Kirchhoff equations and depends on the mode under consideration. Kirchhoff's equations with the assumption in Fig. 1 emf direction and currents have the following form:

$$\left.\begin{aligned}e_a&=-e_A-i_A z_A=-e_A-i_A r_A-x_A\rho i_A\\e_b&=-e_B-i_B z_B=-e_B-i_B r_B-x_B\rho i_B\\e_c&=-e_C-i_C z_C=-e_C-i_C r_C-x_C\rho i_C\end{aligned}\right\}. \quad (14)$$

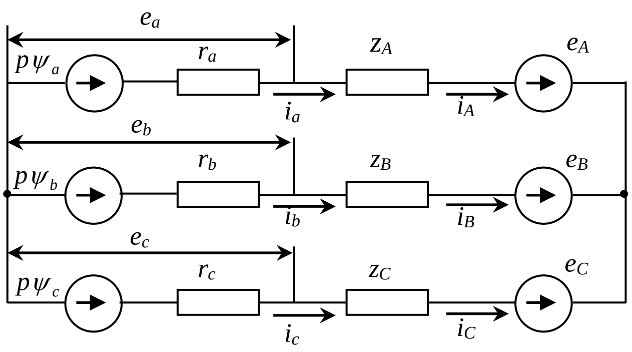

Fig. 1: The direction of emf and currents in system

After substituting (14) into (1), we obtain equations for the analysis of asymmetric modes, taking into account the resistances between the generator and the system.

Phase voltages on buses of infinite power are determined by the well-known expressions:

$$\left.\begin{aligned} e_A &= e\cdot\sin\tau \\ e_B &= e\cdot\sin(\tau-\rho) = -e\left(\frac{1}{2}\sin\tau+\frac{\sqrt{3}}{2}\cos\tau\right) \\ e_C &= e\cdot\sin(\tau+\rho) = -e\left(\frac{1}{2}\sin\tau-\frac{\sqrt{3}}{2}\cos\tau\right) \end{aligned}\right\}. \quad (15)$$

IV. ADDITIONAL RELATIONS FOR CALCULATING SOME ASYMMETRIC MODES

To solve system (1) taking into account (14), it is necessary to establish a connection between the currents of the generator i_a, i_b, i_c and the currents in the buses i_A, i_B, i_C or introduce additional relationships characterizing the operating mode of the synchronous machine.

1. Asymmetry of the resistance system between the generator and the system $Z_A \neq Z_B \neq Z_C$. In this case, as follows from Fig. 1

$$i_a = i_A;\, i_b = i_B;\, i_c = i_C. \quad (16)$$

System of equations (1) taking into account (14) is simplified

$$\left.\begin{aligned} p\psi_a &= -e_A - i_a(r_a + r_A) - x_A p i_a \\ p\psi_b &= -e_B - i_b(r_b + r_B) - x_B p i_b \\ p\psi_c &= -e_C - i_c(r_c + r_C) - x_C p i_c \end{aligned}\right\}. \quad (17)$$

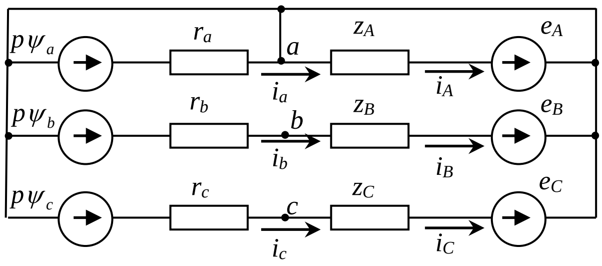

Fig 2: Single-phase generator phase short circuit to common neutral generator-system

2. Single-phase short circuit of the generator phase to the common neutral generator-system. In general, the short circuit mode can be calculated in the presence of resistance asymmetry $Z_A \neq Z_B \neq Z_C$.

Additional relations are obtained from Fig. 2.

$$i_b = i_B;\, i_c = i_C;\, e_a = 0. \quad (18)$$

The system of equations (1) takes the form:

$$\left.\begin{aligned} p\psi_a &= -i_a r_a \\ p\psi_b &= -e_B - i_b(r_b + r_B) - x_B p i_b \\ p\psi_c &= -e_C - i_c(r_c + r_C) - x_C p i_c \end{aligned}\right\}. \quad (19)$$

Short circuit of two phases of the generator to a common neutral generator-system. As follows from Fig. 3, when phases a and b of the generator are shorted to neutral, we obtain the following additional relations:

$$i_c = i_C;\, e_a = 0;\, e_b = 0. \quad (20)$$

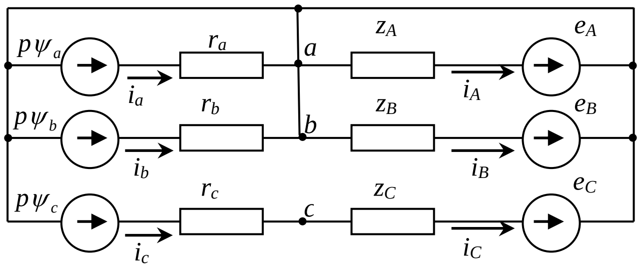

Fig. 3: Short circuit of two phases of the generator to a common neutral generator-system

Taking them into account, system of equations (1) takes the form:

$$\left.\begin{aligned} p\psi_a &= -i_a r_a \\ p\psi_b &= -i_b r_b \\ p\psi_c &= -e_C - i_c(r_c + r_C) - x_C p i_c \end{aligned}\right\}. \quad (21)$$

4. Two-phase short circuit. With a short circuit of phases a and b, we obtain only one obvious relationship from Fig. 4:

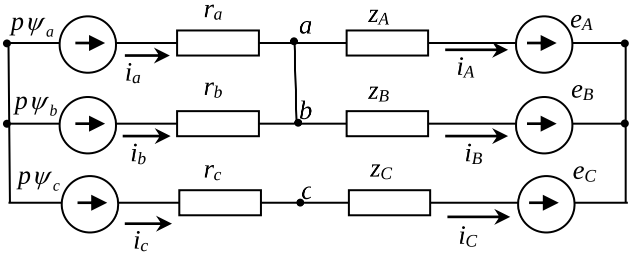

Fig. 4: Two phase short circuit

$$i_c = i_C \quad (22)$$

The other two relations are obtained after introducing an additional assumption:

The values of active and reactive resistances between the short circuit point and infinite power buses in short-circuited phases are equal to

$$Z_A = Z_B = Z_C, \; r_A = r_B = r_C; \, x_A = x_B = x_C.$$

The introduction of this assumption allows us to obtain differential equations for stator circuits, which are easily solved by numerical methods.

From expressions

$$\left.\begin{array}{l} e_a = -e_A - i_A z_A \\ e_b = -e_B - i_B z_A \end{array}\right\}. \quad (23)$$

we obtain expressions for $e_a + e_b$ и $e_a - e_b$:

$$\left.\begin{array}{l} e_a + e_b = -(e_A + e_B) - z_A(i_A + i_B) \\ e_a - e_b = 0 \end{array}\right\}. \quad (24)$$

Using known relations $e_A + e_B + e_C = 0$ and $i_A + i_B + i_C = 0$, get

$$e_A + e_B = -e_C \;\; \text{и} \;\; i_A + i_B = -i_C \quad (25)$$

After substituting (25) into (24) we get

$$e_a + e_b = e_C + z_A i_C;$$
$$e_a - e_b = 0.$$

From here

$$e_a = \frac{e_C}{2} + \frac{z_A}{2} i_C;$$
$$e_b = \frac{e_C}{2} + \frac{z_A}{2} i_C.$$

The system of equations (1) takes the form:

$$\left.\begin{array}{l} p\psi_a = \dfrac{e_C}{2} + \dfrac{x_A \rho i_c}{2} - i_a r_a + \dfrac{i_c r_A}{2}; \\ p\psi_b = \dfrac{e_C}{2} + \dfrac{x_A \rho i_c}{2} + \dfrac{i_c r_A}{2} - i_b r_b; \\ p\psi_c = e_C - x_C \rho i_c - i_c(r_c + r_C). \end{array}\right\} \quad (26)$$

5. Phase failure a.

Phase loss is a special case of asymmetry of the resistance system, when the resistance is $Z_A = \infty$. However, it is not possible to introduce such a resistance value into the CT. We can limit ourselves to including a finite, but sufficiently large value Z_A in the phase. Its value is limited by the scale factors adopted for resistances.

This technique can also be used to consider the operating mode during phase failure. Consider the current in phase a to be equal to zero $i_a = i_A = 0$, which corresponds to the physics of the process, and the flux linkages of phase a, determined by solving the first equation of system (1), are found using the equation

$$\psi_a = i_b x_{ab} + i_c x_{ac} + i_f x_{af} + i_{kd} x_{af} + i_{kq} x_{akq},$$

where $x_{ab}, x_{ac}, x_{af}, x_{akq}$ – mutual reactivity of phase a and the corresponding circuits, determined by the following relations through the known parameters of the machine:

$$\left.\begin{array}{l} x_{ab} = \dfrac{1}{3}x_0 - \dfrac{1}{6}(X_d + X_q) - \dfrac{1}{3}(X_d - X_q)\cos(2\gamma - \rho) \\ x_{ac} = \dfrac{1}{3}x_0 - \dfrac{1}{6}(X_d + X_q) - \dfrac{1}{3}(X_d - X_q)\cos(2\gamma + \rho) \\ x_{af} = x_{ad} \cdot \cos\gamma \\ x_{akq} = x_{aq} \cdot \sin\gamma \end{array}\right\}. \quad (27)$$

6. Single-phase short circuit of the generator to the neutral of the system (Fig. 5) is accepted: $r_a = r_b = r_c$, locked phase a.

We obtain relations for currents

$$i_b = i_B, \, i_c = i_C \quad (28)$$

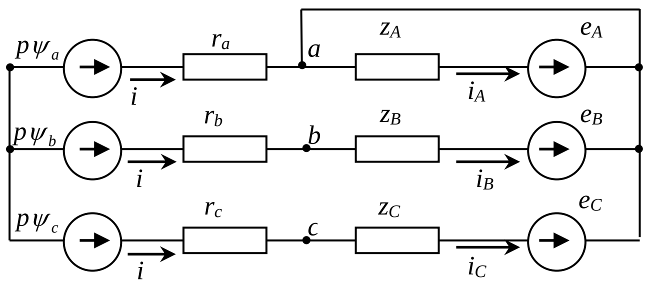

Fig. 5: Phase failure a

To obtain the third necessary relationship, we compose the Kirchhoff equations for stresses:

$$\begin{array}{l} p\psi_a + i_a r_a - p\psi_b - i_b r_b - e_B - i_b z_B = 0; \\ p\psi_a + i_a r_a - p\psi_c - i_c r_c - e_C - i_c z_C = 0. \end{array} \quad (29)$$

Adding equations (29) and using the relations:

$$p\psi_a + p\psi_b + p\psi_c = 0; \quad i_a + i_b + i_c = 0. \quad (30)$$

get:

$$3p\psi_a + 3i_a r_a = e_C + i_c z_C + e_B + i_b z_B,$$

where

$$p\psi_a = \frac{1}{3}\left(e_C + i_c z_C + e_B + i_b z_B\right) - i_a r_a. \quad (31)$$

The equations for phases b and c are obtained similarly:

$$\begin{aligned} p\psi_b &= -\frac{1}{3}\left(2e_B - e_C + 2i_b z_B - i_c z_C\right) - i_b r_B \\ p\psi_c &= -\frac{1}{3}\left(2e_C - e_B + 2i_c z_C - i_b z_B\right) - i_c r_c. \end{aligned} \quad (32)$$

7. We consider a two-phase short circuit of the generator to the system neutral (Fig. 6) to be equal to the active resistance of the generator phases $r_a = r_b = r_c$; Phases a and b are shorted.

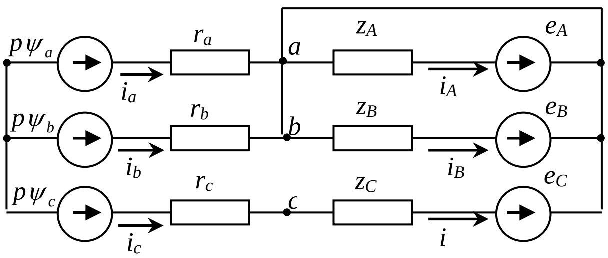

Fig. 6: Single-phase short circuit of the generator to neutral

Composing, as in the previous case, the Kirchhoff equations

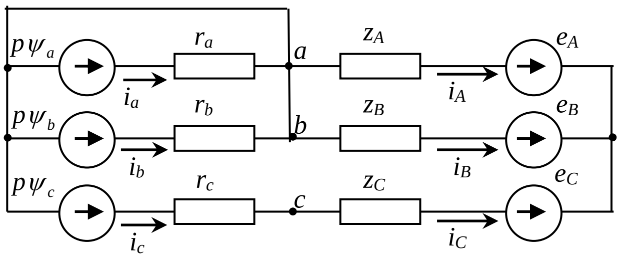

Fig. 8: Two-phase short circuit to generator neutral

$$\left.\begin{aligned} &p\psi_a + i_a r_a - p\psi_c - i_c r_c - e_C - i_c z_C = 0 \\ &p\psi_a + i_a r_a - p\psi_b - i_b r_b = 0 \\ &p\psi_b + i_b r_b - p\psi_c - i_c r_c - e_C - i_c z_C = 0 \end{aligned}\right\} \quad (33)$$

get

$$\left.\begin{aligned} p\psi_c &= -\frac{2}{3}\left(e_C + i_c z_C\right) - i_c r_c \\ p\psi_a &= \frac{1}{3}\left(e_C + i_c z_C\right) - i_a r_a \\ p\psi_b &= \frac{1}{3}\left(e_C + i_c z_C\right) - i_b r_b \end{aligned}\right\}. \quad (34)$$

8. Single-phase short circuit to generator neutral (Fig. 7). Phase a is short-circuited. For phases b and c with a ratio of $i_b = i_B$ and $i_c = i_C$, from the Kirchhoff equations we obtain:

$$\left.\begin{aligned} p\psi_a &= -i_a r_a \\ p\psi_b &= e_A + i_A z_A - e_B - i_b z_B - i_b r_b \\ p\psi_c &= e_A + i_A z_A - e_C - i_b z_C - i_c r_c \\ i_A &= -\left(i_B + i_C\right) \end{aligned}\right\}. \quad (35)$$

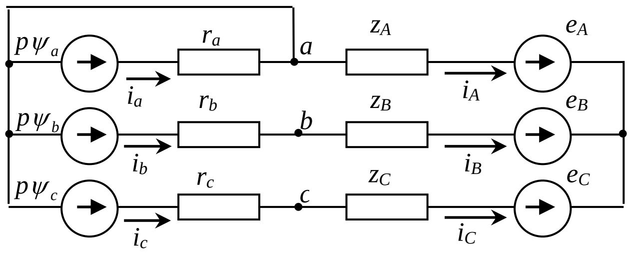

Fig 7: Single-phase short circuit to generator

9. Two-phase short circuit to the neutral of the generator (Fig 8). Phases a and b are closed. For phase c, the relation holds $i_c = i_C$. Compose the Kirchhoff equations (Wood & Wollenberg, 1996) (Yakimchuk, 2000) (Yan & Xu, 2020).

$$\left.\begin{aligned} &p\psi_a = -i_a r_a \\ &p\psi_b = -i_b r_b \\ &i_A + i_B + i_C = 0 \\ &p\psi_c + i_c r_c + e_C + i_C z_C - e_A - i_A z_A = 0 \\ &p\psi_c + i_c r_c + e_C + i_C z_C - e_B - i_B z_B = 0 \end{aligned}\right\}. \quad (36)$$

From the last two equations, under the condition $Z_A = Z_B = Z_C$, we obtain

$$p\psi_c = -\frac{3}{2}\left(e_C + i_C z_C\right) - i_c r_c. \quad (37)$$

Taking into account the reactivity between the generator and the infinite power system led to the appearance of derivatives of the stator phase currents on the right sides of the differential equations of phase voltages.

V. SIMULATION RESULTS OF ASYMMETRIC TRANSIENTS

The results of analytical studies of the considered asymmetrical transient processes (with asymmetrical short circuits) on phase coordinates are summarized in Table 1.

Table 2 and Fig. 9 present the numerical results of computer calculations based on the obtained solutions for transient asymmetrical short-circuit processes in the electrical system. The table data confirms the effectiveness of the proposed approach to studying asymmetric transient processes on phase coordinates.

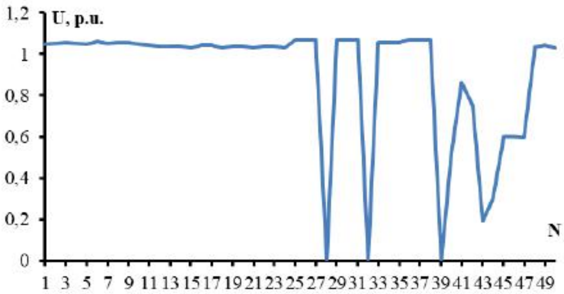

Fig. 9: Voltage profile for a synchronous generator-grid network with asymmetrical short circuits

As can be seen from Table 2 and Fig. 9, in case of short circuits on buses 28, 32 and 39, the voltage on these buses $\dot{U} = 0$ and in consumer nodes 1-9, 25-32 and 36-50 active and reactive loads is zero. On buses 40-47 the voltage is within $(0{,}520 - 0{,}594)U_{nom}$ The results obtained confirmed the pre-modeled modes for the asymmetric transient modes considered above

VI. CONCLUSION

1. Using phase coordinates, a method for calculating asymmetrical transient modes of a synchronous generator operating on an electrical network is presented. As a result of transformations, differential equations of a synchronous machine were obtained in axes a, b, c for stator quantities and axes d, q, 0 for rotor quantities. The variable coefficients included in these equations are functions of $\sin\gamma$ and $\cos\gamma$, therefore their calculation on a CT does not greatly complicate the calculation task compared to the calculation of symmetric modes in the d, q, 0 axes.

2. Equations are given that make it possible to take into account, under certain assumptions, active and inductive resistance, the connection between the generator and the system of infinite power. Based on the conditions characterizing asymmetrical modes, equations for the stator circuits of a generator operating on infinite power systems for various asymmetrical short circuits are obtained.

3. Numerical results of computer calculations based on the obtained solutions for transient asymmetrical short circuit processes in the electrical system are presented. The table data confirms the effectiveness of the proposed approach to studying asymmetric transient processes on phase coordinates.

Table 1: Results of analytical studies of asymmetrical transient processes

Types of asymmetrical short circuits	**System equation (1)**		
	$p\psi_a = e_a - i_a z_a - i_a r_a$	$p\psi_b = e_b - i_b z_B - i_b r_b$	$p\psi_c = e_C - i_c z_C - i_c r_c$
Single-phase short circuit to common neutral	$p\psi_a = -i_a r_a$	$p\psi_b = -e_B - i_b r_b - i_b z_B$	$p\psi_c = -e_C - i_c z_C - i_c r_c$
Two-phase short circuit to common neutral	$p\psi_a = -i_a r_a$	$p\psi_b = -i_b r_b$	$p\psi_c = -e_C - i_c z_C - i_c r_c$
Two-phase short circuit	$p\psi_a = \frac{e_C}{2} - i_a r_a + \frac{i_c z_A}{2}$	$p\psi_b = \frac{e_C}{2} + \frac{i_c z_A}{2} - i_b r_b$	$p\psi_c = -e_C - i_c z_A - i_c r_c$
Single-phase short circuit to system neutral	$p\psi_a = \frac{1}{3}(e_C + i_c z_C +$ $e_B + i_b z_B) - i_a r_a$	$p\psi_b = -\frac{1}{3}(2e_B - e_C +$ $+ 2i_b z_B - i_c z_C) - i_b r_b$	$p\psi_c = -\frac{1}{3}(2e_C - e_B +$ $+ 2i_c z_C - i_b z_B) - i_c r_c$
Two-phase short circuit to system neutral	$p\psi_a = \frac{1}{3}(e_C + i_c z_C) - i_a r_a$	$p\psi_b = \frac{1}{3}(e_C + i_c z_C) - i_b r_b$	$p\psi_c = -\frac{2}{3}(e_C + i_c z_C) - i_c r_c$
Single-phase short circuit to generator neutral	$p\psi_a = -i_a r_a$	$p\psi_b = e_A + i_A z_A -$ $- e_B - i_b z_B - i_b r_b$	$p\psi_c = e_A + i_A z_A - e_C -$ $- i_c z_C - i_c r_c = e_A +$ $+ i_B z_A - e_C - i_c(z_A + z_C) - i_c r_c$

Two-phase short circuit to generator neutral	$p\psi_a = -i_a r_a$	$p\psi_b = -i_b r_b$	$p\psi_c = -\frac{3}{2}(e_C + i_C z_C) - i_c r_c$

Table 2: Calculation results based on the obtained solutions for asymmetrical short circuit

Node No.	Node type	*U* (p.u.)	Angle (deg)	P_g (MW)	Q_g (MVAR)	P_l (MW)	Q_l (MBAP)
1	3	1,050	-1,916	0,000	0,000	0,000	0,000
2	3	1,051	-121,674	0,000	0,000	0,000	0,000
3	3	1,055	116,154	0,000	0,000	0,000	0,000
4	3	1,052	68,661	0,000	0,000	0,000	0,000
5	3	1,050	-30,575	0,000	0,000	0,000	0,000
6	3	1,062	-150,839	0,000	0,000	0,000	0,000
7	3	1,053	87,380	0,000	0,000	0,000	0,000
8	3	1,056	-32,461	0,000	0,000	0,000	0,000
9	3	1,056	-152,688	0,000	0,000	0,000	0,000
10	6	1,046	87,388	0,000	0,000	9,111	5,466
11	6	1,044	-33,011	0,000	0,000	9,089	5,453
12	6	1,037	-152,765	0,000	0,000	8,968	5,361
13	6	1,041	87,096	0,000	0,000	10,833	6,861
14	6	1,039	-33,436	0,000	0,000	10,791	6,834
15	6	1,029	-153,101	0,000	0,000	10,587	6,705
16	6	1,042	87,080	0,000	0,000	12,676	7,965
17	6	1,045	-32,377	0,000	0,000	12,625	7,934
18	6	1,032	-153,073	0,000	0,000	12,428	7,809
19	6	1,040	87,025	0,000	0,000	10,820	6,352
20	6	1,038	-33,469	0,000	0,000	10,778	6,525
21	6	1,029	-153,152	0,000	0,000	10,588	6,706
22	6	1,040	87,014	0,000	0,000	14,420	9,012
23	6	1,038	-33,458	0,000	0,000	14,368	8,980
24	6	1,029	-153,148	0,000	0,000	14,129	8,830
25	1	1,070	0,000	38,839	23,199	0,000	0,000
26	1	1,070	-120,072	32,511	21,553	0,000	0,000

Table 2 (continue)

Node No.	Node type	*U* (p.u)	Angle (deg)	P_g (MW)	Q_g (MVAR)	P_l (MW)	Q_l (MBAP)
27	1	1,070	120,072	37,072	16,920	0,000	0,000
28	5	0,000	0,000	0,000	0,000	0,000	0,000
29	4	1,070	90,856	24,543	11,722	0,000	0,000
30	4	1,070	-29,168	16,229	13,057	0,000	0,000

31	4	1,070	-149,192	19,228	5,183	0,000	0,000
32	5	0,000	0,000	0,000	0,000	0,000	0,000
33	6	1,055	87,412	0,000	0,000	6,680	2,598
34	6	1,058	-32,443	0,000	0,000	6,710	2,610
35	6	1,058	-152,634	0,000	0,000	6,714	2,611
36	4	1,070	85,282	10,083	9,429	0,000	0,000
37	4	1,070	-31,670	8,970	7,914	0,000	0,000
38	4	1,070	-151,694	10,947	7,732	0,000	0,000
39	5	0,000	0,000	0,000	0,000	0,000	0,000
40	3	0,520	-93,147	0,000	0,000	0,000	0,000
41	3	0,863	82,090	0,000	0,000	0,000	0,000
42	3	0,751	-9,572	0,000	0,000	0,000	0,000
43	3	0,192	-72,025	0,000	0,000	0,000	0,000
44	3	0,297	-167,711	0,000	0,000	0,000	0,000
45	3	0,601	67,024	0,000	0,000	0,000	0,000
46	3	0,599	-33,474	0,000	0,000	0,000	0,000
47	3	0,594	-153,152	0,000	0,000	0,000	0,000
48	3	1,034	-122,883	0,000	0,000	0,000	0,000
49	3	1,042	116,762	0,000	0,000	0,000	0,000
50	3	1,032	-3,459	0,000	0,000	0,000	0,000

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