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A robust controller design for a robotic system

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Abstract*— This paper proposes a nonsingular fast sliding mode control based on a fuzzy model applied to a robotic system. The objective of the proposed approach to guarantee the convergence of the system to desired trajectories quickly and in finite time in presence of uncertainties and external disturbances. Simulation results are given to see the obtained performances.

I. INTRODUCTION

Modem industrial systems become more and more complex to modeling and subject to both uncertainties and external disturbances, which makes their controlling difficult.

Sliding mode control can be considered a very popular approach to ensure good tracking performances against external disturbances[1],[2]. Despite its performances sliding mode control suffers from two principal drawbacks: presence of chattering phenomena due to presence of signum function, and time convergence cannot imposed. To overcome the first drawback, many works have been developed. In [3]–[5], the switching signal is smoothed by using a low-pass filter. Authors of [6]–[8] proposed to use an adaptive fuzzy system to substitute the switching control and, hence, to eliminate the chattering phenomenon. Other techniques have been developed in the literature. However, this improvement needs a trade off between the smoothness of the switching signal and tracking performances. Second order sliding mode control have been also presented a good solution to chattering but the design procedure is complex and the requires a good knowledge of the studied system [9]–[11].

Other improvements of classical sliding mode control like terminal sliding mode control have been developed, where a non linear surface is used [12], [13]. However, these kinds of controllers suffer from singularity problem due to presence of terms with negative fractional powers [14]. This problem can resolved by using a nonsingular terminal sliding mode controller [15], [16] Nevertheless, this improvement was obtained at the expense of the convergence time which becomes slower. Nonsingular fast terminal sliding mode controller have been developed to overcome singularity and to obtain fast convergence time [17], [18].

Thus, in this paper, we propose a nonsingular fast terminal sliding mode controller for a robotic system which guarantees finite-time convergence, fast speed when the states are far from the origin, avoidance of singularity and without chattering. The control is developed using a fuzzy nominal model witch avoids using approximating system dynamics [19]–[21].

The remainder of this paper is organized as follows : In Section 2, problem statement of controlling a robotic system is treated. Section 3 is dedicated to the controller design and stability analysis. Simulation and results are given in Section 4 to show the effectiveness of the proposed approach. Finally, the conclusion is provided.

II. ROBLEM STATEMENT

Let us consider the dynamic equation of n degree-of-freedom robotic manipulators as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q,\dot{q}) = \Gamma(t) + \Gamma_{ext}(t) \quad (1)$$

Where:

q, $\dot{q}$ and $\ddot{q} \in \mathbb{R}^n$ are the vector of joint position, joint velocity, and joint acceleration, respectively.

$M(q) \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite inertia matrix,

$C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix of centrifugal and Coriolis forces,

$G(q) \in \mathbb{R}^n$ is the vector of gravitational forces,

$\Gamma(t) \in \mathbb{R}^n$ is the vector of input joint torque and $\Gamma_{ext}(t) \in \mathbb{R}^n$ is the vector of unknown external disturbances.

For practical applications, it is impossible to know the exact dynamic model of the robotic manipulators. Hence, the above dynamic quantities can be expressed as:

$$\begin{aligned} \mathbf{M(q)} &= \mathbf{M_0(q)} + \Delta\mathbf{M(q)} \\ \mathbf{C(q,\dot{q})} &= \mathbf{C_0(q,\dot{q})} + \Delta\mathbf{C(q,\dot{q})} \\ \mathbf{G(q)} &= \mathbf{G_0(q)} + \Delta\mathbf{G(q)} \end{aligned} \quad (2)$$

Where:

$M_0(q)$, $C_0(q,\dot{q})$, $G_0(q)$ are the nominal values of $M(q)$, $C(q,\dot{q})$, $G(q)$ respectively and

$\Delta M(q)$, $\Delta C(q,\dot{q})$, $\Delta G(q)$ are the uncertain parts of $M(q)$, $C(q,\dot{q})$, $G(q)$ respectively.

Using equation (2), the dynamic model of the robotic manipulators can be expressed as:

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q,\dot{q}) = \Gamma(t) + \delta(q,\dot{q},\ddot{q}) \quad (3)$$

Where:

$$\delta(q,\dot{q},\ddot{q}) = \Gamma_{ext}(t) - \Delta M(q)\ddot{q} - \Delta C(q,\dot{q})\dot{q} - \Delta G(q)$$

Let define the tracking error $e = q - q_d$ and its time derivative $\dot{e} = \dot{q} - \dot{q}_d$ where q_d the desired trajectory. Then the error dynamic of the robotic manipulators with the uncertainties and disturbances can be written as:

$$\ddot{e} = f(e,\dot{e}) + g(e,\dot{e})\Gamma(t) + D(e,\dot{e}) \quad (4)$$

Where

$$f(e,\dot{e}) = -M_0^{-1}(q)[C_0(q,\dot{q})\dot{q} + G_0(q,\dot{q}) - \ddot{q}_d,$$

$$g(e,\dot{e}) = M_0^{-1}(q) \ and$$

$$D(e,\dot{e}) = M_0^{-1}(q)\,\delta(q,\dot{q},\ddot{q})$$

As given in [14], the upper bound of lumped uncertainty can be expressed as:

$$|D(e,\dot{e})| \leq a_0 + a_1|q| + a_2|\dot{q}|^2 \quad (5)$$

Where b_0, b_1 and b_2 are positive scalars.

The next task is to develop a robust controller based on nonsingular fast terminal sliding mode control allowing to tracking objectives.

III. CONTROLLER DESIGN

To design our controller, let consider the following nonsingular terminal sliding surface:

$$S(t) = e + k_1|e|^{\alpha} sign(e) + k_2|\dot{e}|^{\beta} sign(\dot{e}) \quad (6)$$

Where k_1 and k_2 are positive constants,

$1 < \beta < 2$ and $\alpha > \beta$.

The structure of this surface allows us to attain fast convergence of the tracking error to zero. Indeed, if the position initial value is far from the desired one, then the term $k_1|e|^{\alpha}sign(e)$ will be dominant, which leads to a fast convergence. In the case where the system is near the desired trajectory, the term $k_2|\dot{e}|^{\beta} sign(\dot{e})$ must ensuring a finite time convergence.

The time derivative of the sliding surface can be written as:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.\ddot{e} \quad (7)$$

Our control law will be composed from two terms. The first one, named equivalent control $\Gamma_e(t)$, is dedicated to maintain the system on the sliding surface. The second term, $\Gamma_s(t)$ called switching signal, must force the system to converge to the sliding surface. Then, to design the equivalent control law $\Gamma_e(t)$, we consider that the system is on the surface ($S(t) = 0$) and remains on ($\dot{S}(t) = 0$). In this case, the system is considered insensitive to uncertainties and external disturbances [1].

Using (4) equation (7) can be rewritten as:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.[f(e,\dot{e}) + g(e,\dot{e})\Gamma_e(t)] \quad (8)$$

Then the expression of equivalent control law can be expressed as:

$$\Gamma_e(t) = -g^{-1}(e,\dot{e}).[f(e,\dot{e}) + [\beta.k_2]^{-1}|\dot{e}|^{2-\beta}(1 + \alpha.k_1|e|^{\alpha-1})\,sign(\dot{e})] \quad (9)$$

Note that, we used the fact that $\dot{e} = |\dot{e}|.sign(\dot{e})$ to writ equation (9) in a compact form.

Our next task is to determine the expression of the switching signal $\Gamma_s(t)$ allowing to force the system to reach the sliding surface in presence of uncertainties and external disturbances.

In this case, equation (7) becomes:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.[f(e,\dot{e}) + g(e,\dot{e})\Gamma(t) + D(e,\dot{e})] \quad (10)$$

Using (9), we can rewrite (10) as:

$$\dot{S}(t) = \dot{e} + \alpha\cdot k_1|e|^{\alpha-1}\dot{e} + \beta\cdot k_2|e|^{\beta-1}\cdot [f(e,\dot{e}) + g(e,\dot{e})\Gamma_s(t)]\beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (11)$$

According to the definition of the equivalent control, equation (11) can be simplified to:

$$\dot{S}(t) = \beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (12)$$

To deduce the expression of $\Gamma_s(t)$ allowing the switching condition, we consider the following Lyapunov function:

$$V(t) = \frac{1}{2}S^2(t) \quad (13)$$

Differentiating $V(t)$ with respect to time and using (12) lead to:

$$\dot{V}(t) = S(t).\beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (14)$$

Choosing $\Gamma_s(t)$ as:

$$\Gamma_s(t) = -g^{-1}(e,\dot{e})[k_{01}.S(t) + (k_{02} + a_0 + a_1|q| + a_2|\dot{q}|^2).sign(S(t))] \quad (14)$$

Where k_{01} and k_{02} are two positive scalars.

The time derivative of the Lyapunov function becomes:

$$\dot{V}(t) = S(t)\cdot\beta\cdot k_2|\dot{e}|^{\beta-1}\cdot[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})]$$

$$= \beta.k_2|\dot{e}|^{\beta-1}.[-k_{01}.S^2(t) - (k_{02} + a_0 + a_1|q| + a_2|\dot{q}|^2).|S(t)| + D(e,\dot{e})] \quad (15)$$

Using the assumption (5), we obtain the following inequality:

$$\dot{V}(t) \le \beta.k_2|\dot{e}|^{\beta-1}.[-k_{01}.S^2(t) - k_{02}.|S(t)|] \le 0 \quad (16)$$

Based on the Lyapunov theorem, the system converges asymptotically to the sliding surface and remains on.

To prove convergence in finite time, let us take up inequality (17):

$$\dot{V}(t) \le -\beta.k_{01}.k_2|\dot{e}|^{\beta-1}.S^2(t) - \beta.k_{02}.k_2|\dot{e}|^{\beta-1}.|S(t)| \quad (18)$$

$$\dot{V}(t) = \frac{dV(t)}{dt} \le -\underbrace{2.\beta.k_{01}.k_2|\dot{e}|^{\beta-1}}_{\beta_1}.V(t) - \underbrace{\sqrt{2}\beta.k_{02}.k_2|\dot{e}|^{\beta-1}}_{\beta_2}.V^{\frac{1}{2}}(t) \quad (17)$$

Then we can obtain:

$$dt \le \frac{-dV(t)}{\beta_1.V(t)+\beta_2.V^{\frac{1}{2}}(t)} = -2.\frac{dV^{\frac{1}{2}}(t)}{\beta_1.V^{\frac{1}{2}}(t)+\beta_2} \quad (18)$$

If we consider that the system converges to 0 at $t = t_r$ implies that:

$$\int_0^{t_r} dt \le \int_{V(0)}^{V(t_r)} \frac{-2.dV^{\frac{1}{2}}(t)}{\beta_1.V^{\frac{1}{2}}(t)+\beta_2} = \left[-\frac{2}{\beta_1}ln\left(\beta_1 V^{\frac{1}{2}}(t) + \beta_2\right)\right]_{V(0)}^{V(t_r)} \quad (19)$$

Hence,

$$t_r \le \frac{2}{\beta_1}ln\left(\frac{\beta_1 V^{\frac{1}{2}}(0)+\beta_2}{\beta_2}\right) \quad (20)$$

Consequently, the control law $\Gamma(t) = \Gamma_e(t) + \Gamma_s(t)$, whose terms are defined by equations (9) and (15), guarantees the asymptotic stability of the closed loop system and the convergence of the tracking error in a finite time.

IV. FIGURES SIMULATION AND RESULTS

To show the performances of the performances of the proposed approach, we consider a two-link robot, shown in figure 1, whose dynamics equation is given by [15]:

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix} = \begin{bmatrix} \Gamma_1(t) \\ \Gamma_2(t) \end{bmatrix} + \begin{bmatrix} \Gamma_{ext1}(t) \\ \Gamma_{ext2}(t) \end{bmatrix}$$

Where:

$$M_{11}(q) = (m_1 + m_2)l_1^2$$

$$M_{12}(q) = M_{21}(q) = m_2 l_1 l_2(\sin(q_1)\sin(q_2) + \cos(q_1)\cos(q_2))$$

$M_{22}(q) = m_2 l_2^2$

$C_{11}(q,\dot{q}) = -m_2 l_1 l_2(\cos(q_1)\sin(q_2) - \sin(q_1)\cos(q_2))\dot{q}_2$

$C_{21}(q,\dot{q}) = -m_2 l_1 l_2(\cos(q_1)\sin(q_2) - \sin(q_1)\cos(q_2))\dot{q}_1$

$C_{11}(q,\dot{q}) = C_{22}(q,\dot{q}) = 0$

$G_1(q) = -(m_1 + m_2) l_1 g.\sin(q_1)$

$G_2(q) = -m_2 l_2 g.\sin(q_2)$

$m_1 = m_2 = 1Kg;\ l_1 = l_2 = 1m;\ g = 9.8ms^{-2}$

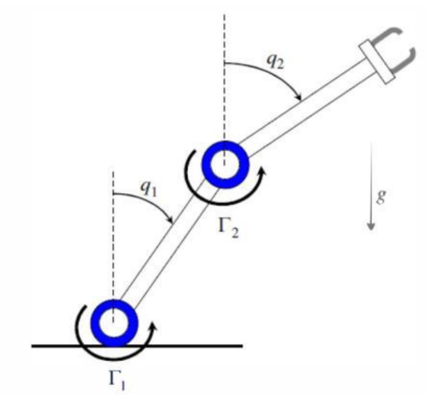

Fig.1: Two link robot manipulators.

To construct the type 2 fuzzy nominal model, we consider that the positions q_1 and q_2 are constrained within $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$, witch leads to nine fuzzy rules. Each one of them gives the relation between the equilibrium point and the corresponding local model. Then, each rule uses a type 2 fuzzy sets in the antecedent part to describe the equilibrium point and the consequent part the corresponding local model. Using the product as an interference engine, the method of center set for the reduction type and center of gravity for defuzzification, the output fuzzy system will be giving the type 2 fuzzy nominal model [19] [22].

From Figure 2, we can see that the system converges to the desired trajectories quickly and achieves good tracking performance. Furthermore, both the position and velocity tracking errors tend to zero after a short transient due to errors in initial conditions (figure 3). Figure From 5 shows the applied torques with smooth variation without chattering. Phase portraits of figure 4 illustrate that system states reach the sliding surface in finite-time, and then they converge to 0 along the pre-described surface. Thus, we can conclude that the proposed approach ensures high tracking precision, fast response, singularity avoidance and strong robustness to external disturbances and modeling uncertainties[9], [18].

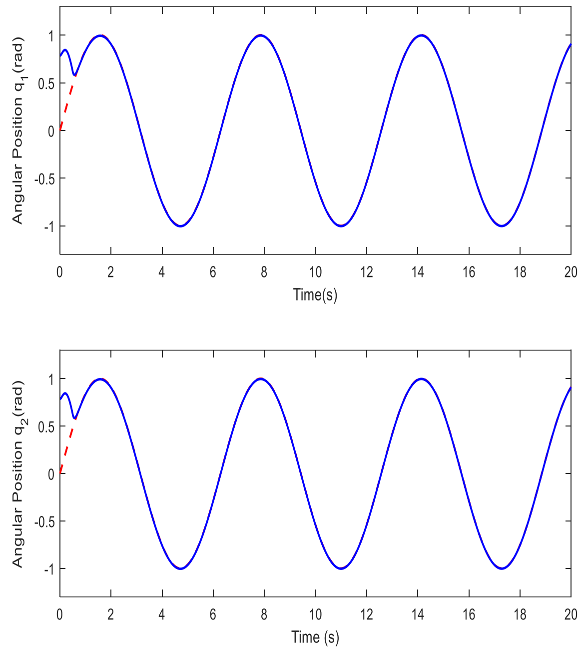

Fig.2: Angular position Tracking.

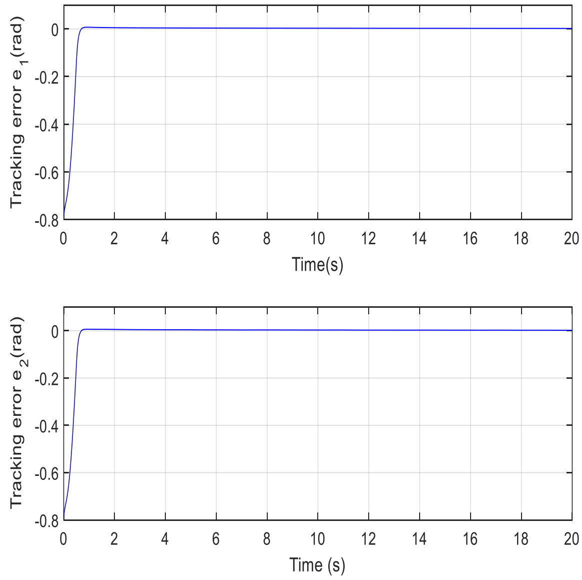

Fig.3: Angular position tracking error.

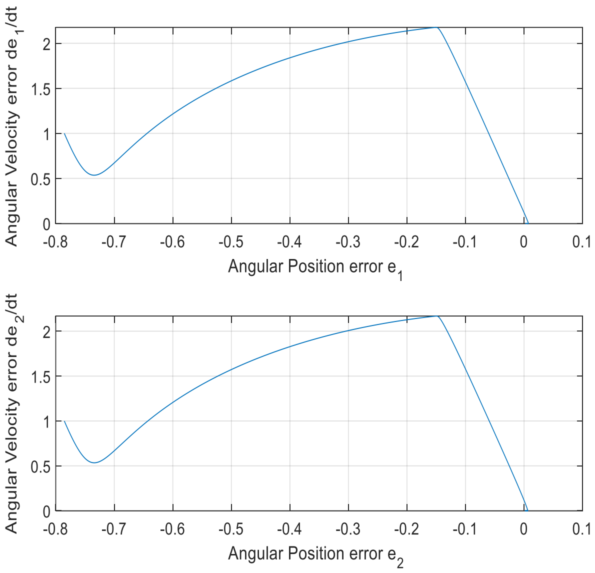

Fig.4: Phase portrait of two joints.

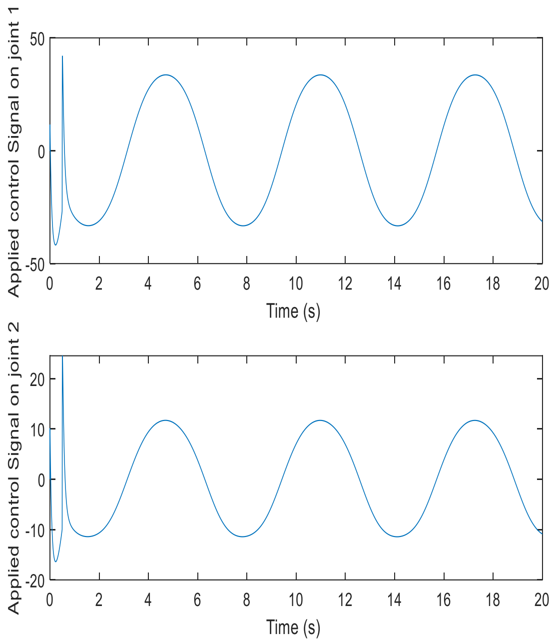

Fig.5: Applied control signals.

V. CONCLUSION

In this work, a nonsingular fast sliding mode control based on a fuzzy model for robotic systems is presented. Using a type fuzzy model allows to obtain a simple nominal model to simplify the control law deduction. This later has been developed such that it guarantees the convergence of the system to desired trajectories quickly and in finite time. Simulation results have been given to show the tracking performances (convergence to the desired trajectory and finite time convergence) despite the presence of modeling uncertainties and external disturbances. As perspective of this work to improve the control law design by reducing number of used parameters.

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