

Research on Control Scheme Based on Ball & Beam System

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Keywords — *Ball & Beam system, Fuzzy PID (Proportion Integration Differentiation) controller, Lagrangian equation, Stability, Simulink simulation*

Abstract — *In the research of control theory, the open-loop unstable Ball & Beam system is one of the typical research objects. This article studies the PID (Proportion Integration Differentiation) controller design method and fuzzy PID controller design method based on the Ball & Beam system, aiming to achieve effective control of system stability and performance. Firstly, a force analysis was conducted on the Ball & Beam system, and a mathematical model was established based on the Lagrangian equation, providing a foundation for the subsequent design of control strategies. Next, we conducted an analysis of stability and performance indicators, including overshoot, adjustment time, etc., to evaluate the performance of the two PID controllers. Finally, the optimal PID controller is determined by adjusting their respective control parameters, and the accuracy, speed, and stability of the proposed algorithm and model are verified through their corresponding response curves. The simulation and experimental results show that the fuzzy PID controller can quickly stabilize the ball at the target position compared to the conventional PID controller and has good practicality, stability, rapidity, and accuracy.*

I. INTRODUCTION

The Ball & Beam system is teaching and experimental equipment designed for basic control courses in automation, mechanical electronics, electrical engineering, and other majors. The system covers many classic and modern design methods [1–4]. This system has

a very important property, which is that it is open-loop unstable. Specifically, in the absence of feedback control, when the system is slightly disturbed, the system will diverge, meaning that the crossbar will rotate in one direction, leave the equilibrium position, and cannot return to the equilibrium position. Under the action of gravity, the

ball will accelerate and roll along the crossbar to the end, or even leave the crossbar. Due to the instability of the open loop, it is necessary to design a controller to achieve the goal of controlling the ball.

The control problem of unstable systems is a challenge that most control systems need to overcome, and it is necessary to study it. The Ball & Beam system is the best experimental tool used to solve this contradiction due to its simple structure, intuitive and clear understanding,

safety, and the important dynamic characteristics of an unstable system. Which can meet the experimental requirements of courses such as automatic control principles and modern control engineering [5, 6]. The Ball & Beam system consists of a control computer, servo driver, photoelectric encoder, linear displacement sensor, servo motor, and ball and rod device, forming a closed-loop system (Figure 1).

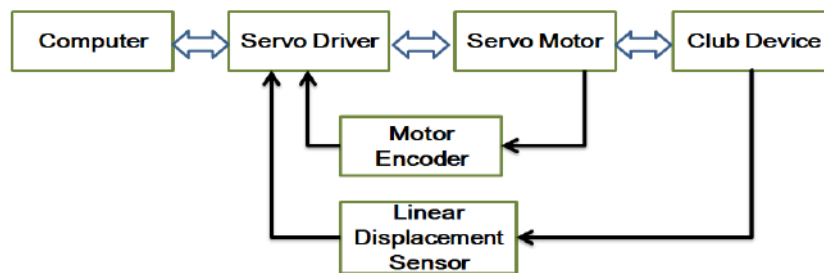


Fig.1. Schematic Diagram of Ball & Beam System Composition

The photoelectric encoder communicates with the computer through an interface regarding the angle and angular velocity between the lever arm and the horizontal direction. In the control system, the control position and parameters of the ball are input, and the motor rotation direction, rotation speed, acceleration, etc. are calculated and output through control decisions. The corresponding control quantities are generated by the servo driver, which sends out signals to rotate the motor and drive the crossbar to move, thereby controlling the position of the ball. This system is a single input (motor angle) and single output (ball position) system. Among them, the motor angle is detected in real-time by the angle encoder of the servo motor, and the position of the ball is detected in real-time by the linear displacement sensor on the track. The control system obtains the real-time motion status of the ball and club system through two detection signals, namely the motor angle and the ball position, thereby achieving control [7].

The Ball & Beam system mainly consists of two parts: control and execution (Figure 2), where the execution part is composed of a base, a ball, a crossbar, a reduction pulley, a support part, and an electric motor. It is a typical four-link mechanism [8, 9]. The crossbar is composed of a stainless steel rod with a scale and a linear displacement sensor. One end of the crossbar is fixed, and the other end is connected to the deceleration belt wheel through a lever arm, which can rotate around its left pivot point. The linear displacement sensor is placed in the groove of the crossbar and can detect the position of the ball in real-time during the control process. When the crossbar is tilted at an angle to the horizontal plane, the ball will roll along the crossbar due to gravity. By controlling the angle between the crossbar and the horizontal plane, the position of the small ball on the crossbar can be controlled, and the control of the crossbar angle is achieved through the output of a Direct Current (DC) servo motor and transmission deceleration. This system is an ideal experimental model for control system design. It is precisely because the

structure of the system is relatively simple that it is easier to understand the control process of the model.

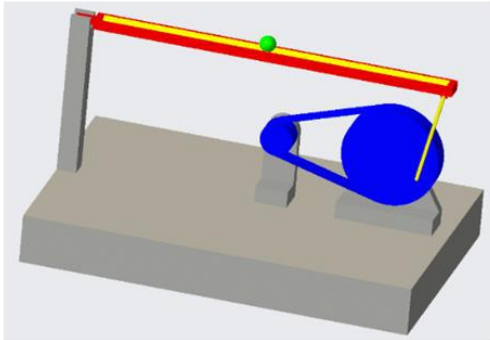


Fig.2 Ball & Beam System

The control system adopts a PID controller for controlling motors [10–12]. The position of the small ball

is collected through a linear displacement sensor, and the controller calculates the control amount based on the position error to control the angle of the motor shaft, thereby controlling the angle of the crossbar and stabilizing the small ball to the target position. By designing a feedback control system to regulate the rotation of the DC motor, the position of the ball on the track can be controlled, ultimately causing the ball to stop at the set position. The transformative impact of the PID control algorithm in this study on the Ball & Beam control system has been deeply explored and analyzed. And focus on using mathematical modeling and Simulink to intuitively demonstrate the controllability of Ball & Beam control systems.

II. MATHEMATICAL FORMULAS

According to the Lagrangian equation [1,2,3,4-6,7,10]:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = - \frac{\partial U}{\partial q_k} \quad (1)$$

Among them, q_k represents generalized coordinates, which are independent parameters that determine the position of the particle system. This study takes α 、 γ as the generalized coordinates of the system (Figure 3), respectively representing the position of the slider and the position of the ball, which are independent of each other but closely connected. As the position α changes, γ follows changes, and the ball also moves accordingly. γ changes with α , and these two parameters determine the position of the ball, so they are chosen as generalized coordinates. After determining the generalized coordinates, it is necessary to analyze the kinetic energy T and potential energy U in the Lagrangian equation. The kinetic energy of a translational object is related to its mass and velocity, and if the object is still rotating, its moment of inertia also needs to be considered. The kinetic energy of an object during translational motion is

$$T = \sum_{i=1}^n \frac{1}{2} m_i V_i^2 \quad (2)$$

The kinetic energy of an object during rotation is

$$T = \frac{1}{2} J W_1^2 \quad (3)$$

When analyzing the ball, the relevant parameters can be determined from Figure 3 (Table 1)

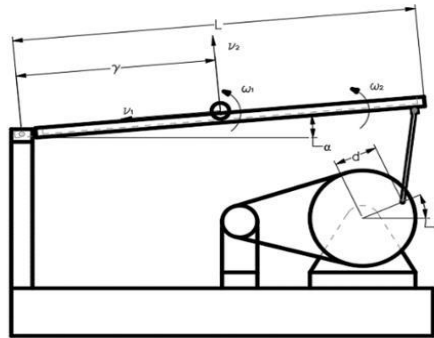


Fig.3 Stress Analysis of Ball & Beam System

Table 1 Ball & Beam System Parameters and Physical Meanings

Variable	Physical Meaning	Parameter Values
m	Ball mass	28g
M	Cross bar mass	
R	Ball radius	6.5mm
g	Gravitational acceleration	10N/kg
L	Cross bar length	150mm
γ	The position of the ball on the crossbar	
d	Distance from gear center to connecting rod point	40mm
J_w	Rotational inertia of the crossbar around the fixed end	
α	Cross bar corner	
θ	Gear angle	
J_b	Moment of inertia of a small ball	

Small ball velocity along the crossbar

$$v_1 = -\dot{\gamma} \tag{4}$$

Ball rotation angular velocity

$$\omega_1 = \frac{v_1}{R} = \frac{-\dot{\gamma}}{R} \tag{5}$$

Cross bar rotation angular velocity

$$\omega_2 = \dot{\alpha} \tag{6}$$

The velocity of a small ball in the direction perpendicular to the crossbar

$$v_2 = \omega_2 \gamma = \gamma \dot{\alpha} \tag{7}$$

From the above formulas, it can be seen that the energy of a small ball is composed of the energy of rolling downward, the energy of the sliding rod pushing the ball, and the energy of the ball rolling itself. The energy expression of the small ball can be listed as formula (8)

$$T_{ball} = \frac{1}{2}mV_1^2 + \frac{1}{2}J_b\omega_1^2 + \frac{1}{2}mV_2^2 \quad (8)$$

Which J_b represents the rotational inertia of the ball itself:

$$J_b = \frac{2}{5}mR^2 \quad (9)$$

Bringing equations (4), (5), and (7) into equation (8) yields

$$\begin{aligned} T_{ball} &= \frac{1}{2}m\dot{\gamma}^2 + \frac{1}{2}J_b \frac{\dot{\gamma}^2}{R} + \frac{1}{2}m\gamma^2\dot{\alpha}^2 \\ &= \frac{1}{2}\left(m + \frac{J}{R^2}\right)\dot{\gamma}^2 + \frac{1}{2}m\gamma^2\dot{\alpha}^2 \end{aligned} \quad (10)$$

The potential energy U of a small ball (with the left support point of the crossbar as the reference point) is expressed as:

$$U = mg\gamma \sin \alpha \quad (11)$$

To calculate the Lagrangian equation, first establish the equation using γ as the generalized coordinate:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\gamma}}\right) - \frac{\partial T}{\partial \gamma} = -\frac{\partial U}{\partial \gamma} \quad (12)$$

$$\frac{\partial T}{\partial \gamma} = \frac{\partial}{\partial \dot{\gamma}}\left[1\frac{1}{2}\left(m + \frac{J_b}{R^2}\right)\dot{\gamma}^2 + \frac{1}{2}m\gamma^2\dot{\alpha}^2\right] = m\dot{\gamma}\dot{\alpha}^2 \quad (13)$$

$$\frac{\partial T}{\partial \dot{\gamma}} = \frac{\partial}{\partial \dot{\gamma}}\left[1\frac{1}{2}\left(m + \frac{J_b}{R^2}\right)\dot{\gamma}^2 + \frac{1}{2}m\gamma^2\dot{\alpha}^2\right] = \left(m + \frac{J_b}{R^2}\right)\dot{\gamma} \quad (14)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\gamma}}\right) = \left(m + \frac{J_b}{R^2}\right)\ddot{\gamma} \quad (15)$$

$$\frac{\partial U}{\partial \gamma} = mg \sin \alpha \quad (16)$$

Substituting equations 13-16 into equation 12 yields

$$\left(m + \frac{J_b}{R^2}\right)\ddot{\gamma} - m\dot{\gamma}\dot{\alpha}^2 + mg \sin \alpha = 0 \quad (17)$$

Establishing equations with α as the generalized coordinate

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) - \frac{\partial T}{\partial \alpha} = -\frac{\partial U}{\partial \alpha} \quad (18)$$

$$\frac{\partial T}{\partial \alpha} = 0 \quad (19)$$

$$\frac{\partial T}{\partial \dot{\alpha}} = m\gamma^2\dot{\alpha} \quad (20)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = m\gamma^2\ddot{\alpha} \quad (21)$$

$$\frac{\partial U}{\partial \alpha} = mg\gamma \cos \alpha \quad (22)$$

Substituting equations 19, 21, and 22 into equation 18 yields

$$\begin{aligned} m\gamma^2\ddot{\alpha} &= -mg\gamma \cos \alpha \\ \gamma\ddot{\alpha} + g \cos \alpha &= 0 \end{aligned} \quad (23)$$

Comparing Equation 17 with Equation 23, it was found that Equation 23 considers fewer parameters and may have significant errors. Therefore, Equation 17 was used to calculate the equation

$$\left(m + \frac{J_b}{R^2}\right)\ddot{\gamma} - m\gamma\dot{\alpha}^2 + mg \sin \alpha = 0 \quad (24)$$

There are also

$$\alpha = \frac{d}{L} \theta \quad (25)$$

Due to the small actual friction force, the friction force is ignored, and since $\dot{\alpha}$ is small, the influence of this term can be ignored. The basic mathematical model is transformed into the following equation

$$mg \sin \alpha = -\left(m + \frac{J_b}{R^2}\right)\ddot{\gamma} \quad (26)$$

When $\alpha \ll 1$ linearize the above equation to obtain the transfer function as follows

$$\frac{\gamma(s)}{\alpha(s)} = \frac{m}{-\left(m + \frac{J_b}{R^2}\right)} \cdot \frac{g}{s^2} \quad (27)$$

However, in the actual control process, the elevation angle α of the rod is achieved by the angle output of the motor. The main factors affecting the relationship between motor angle θ and rod elevation angle α are the reduction ratio and nonlinearity of the belt drive, so we further simplify the model.

$$\alpha(s) = \frac{d}{L} \theta(s) \quad (28)$$

By substituting Equation 27 into Equation 28, we can obtain another model

$$\frac{\gamma(s)}{\theta(s)} = \frac{m}{-\left(m + \frac{J_b}{R^2}\right)} \cdot \frac{g}{s^2} \cdot \frac{d}{L} \quad (29)$$

Substitute the parameters of the Ball & Beam system into Equation 29 to obtain the open-loop transfer function of the system as

$$G(s) = \frac{\gamma(s)}{\theta(s)} = \frac{m}{-\left(m + \frac{5}{R^2}\right)} \cdot \frac{40}{150} \cdot \frac{10}{s^2} = \frac{1}{-\left(1 + \frac{5}{2}\right)} \times \frac{4}{15} \times \frac{10}{s^2} = -\frac{40}{21s^2} \quad (30)$$

Therefore, the Ball & Beam system is a second-order system.

III. SIMULATION OF BALL & BEAM SYSTEM

3.1 Stability Analysis of Ball & Beam System

Analyze the open-loop transfer function of Ball & Beam system (Figure 4)

$$G(s) = -\frac{40}{21s^2} \tag{31}$$

Figure 5 shows the system input signal, while Figure 6 shows the inherent system response as a sine wave function, which is a periodic oscillation system and unstable. Therefore, a PID controller can be added to the front of this system to stabilize it.

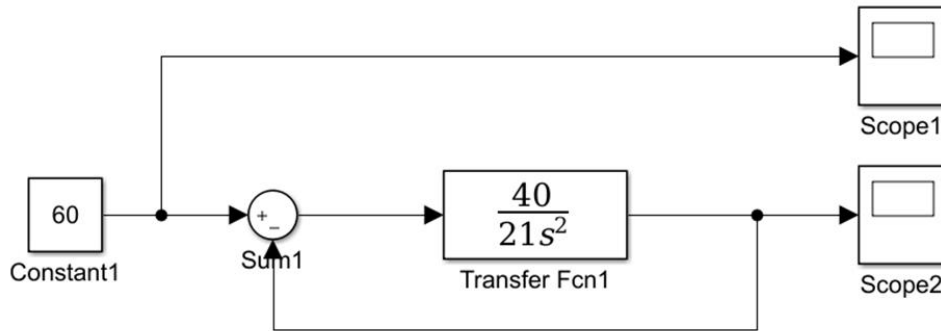


Fig.4 Ball & Beam System Inherent System Simulation Schematic Diagram

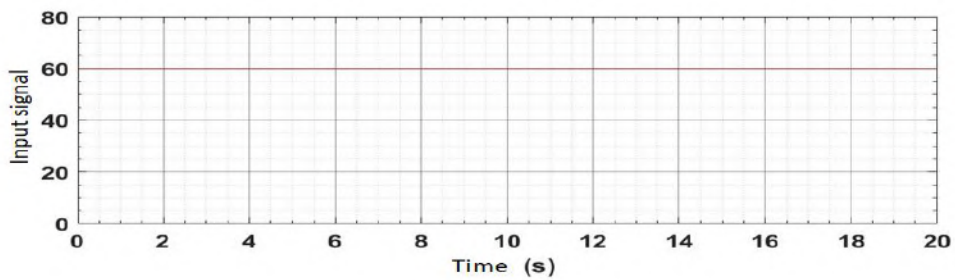


Fig.5 Input Signal of Ball & Beam System

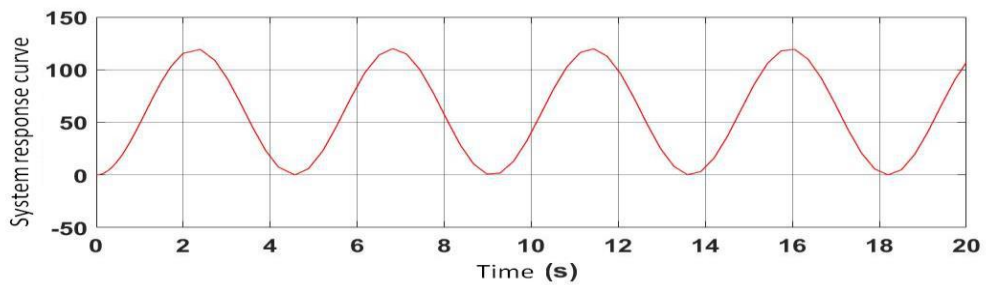


Fig.6 Output Response of Ball & Beam System

3.2 PID Control Principle of Ball & Beam System

3.2.1 Traditional PID Control Principle

In analog control systems, the most commonly used control law for controllers is PID control. The schematic diagram of the traditional PID control system is shown in Figure 7. The system consists of a simulated PID controller and a controlled object.

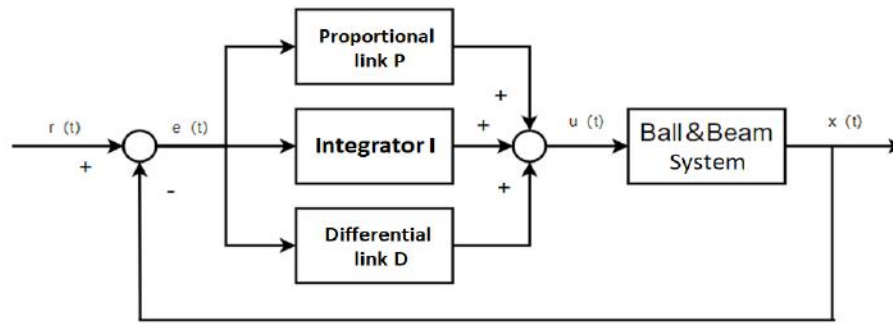


Fig.7 PID Control System Principle Schematic Diagram

PID controller is a linear controller that forms a control deviation based on the given $r(t)$ and the actual output value $u(t)$:

$$e(t) = r(t) - u(t) \tag{32}$$

The proportion (P), integral (I), and derivative (D) of the deviation are linearly combined to form a control variable, which controls the Ball & Beam model, hence the name PID controller. The control law is

$$u(t) = K_p [e(t) + \frac{1}{T} \int_0^t e(t) dt + T_D \frac{de(t)}{dt}] \tag{33}$$

or written in the form of a transfer function

$$G(s) = \frac{U(s)}{E(s)} = K_p (1 + \frac{1}{T_I s} + T_D s) \tag{34}$$

Among them, K_p , T_I , and T_D are proportional constants, integral time constants, and differential time constants, respectively. The function of each correction link of the PID controller is 1. Proportional link: timely and proportionally reflect the deviation function $e(t)$ of the control system. Once a deviation occurs, the controller immediately exerts control action to reduce the deviation. 2. Integration stage: mainly used to eliminate static errors and improve the system's error-free performance. The strength of the integration effect depends on the integration time constant T_I . The larger the T_I , the weaker the integration effect, and vice versa. 3. Differential link: It can reflect the trend (rate of change) of the deviation signal and introduce an effective early correction signal into the system before the deviation signal value becomes too large, thereby accelerating the system's motion speed and reducing adjustment time.

3.2.2 Fuzzy PID Control Principle

The fuzzy PID controller takes error e and error change ec as inputs (Figure 8) to achieve the requirements of fuzzy self-tuning of PID parameters at different times e and ec , and then uses fuzzy control rules to modify the PID parameters online, forming a fuzzy PID controller [13-15].

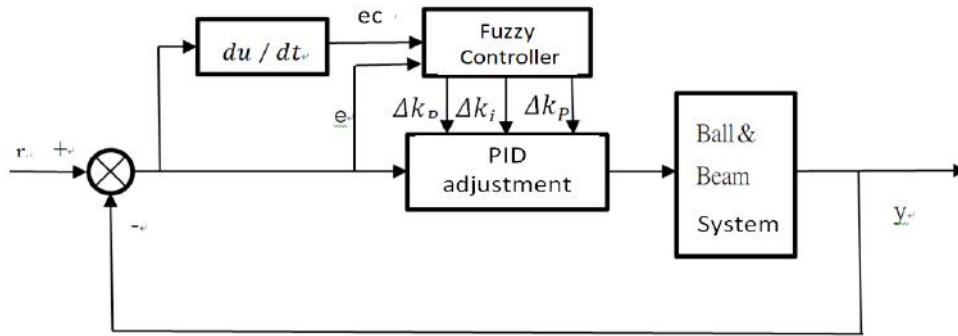


Fig.8. Principle Schematic Diagram of Fuzzy PID Control System

PID parameter fuzzy self-tuning is to identify the fuzzy relationship between the three parameters of PID and the error (e) and error change rate (ec). The controller continuously measures the error (e) and error change rate (ec) during operation, and adjusts the three parameters of PID online according to fuzzy control rules to cope with the deterioration of control performance caused by changes in system operating characteristics, thereby ensuring good static and dynamic performance of the system.

3.3 Ball & Beam System PID Controller Design

3.3.1 Design of Traditional PID Controller

The PID open-loop transfer function is shown in Figure 9.

$$G(s) = \frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_D s \right) \tag{35}$$

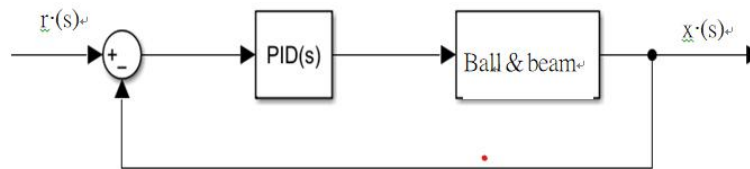


Figure 9 Schematic Diagram of Ball & Beam System PID Controller

The unit closed-loop transfer function can be obtained

$$G_1(s) = \frac{x(s)}{r(s)} = \frac{PID \cdot Ball \& Beam}{1 + PID \cdot Ball \& Beam} \tag{36}$$

Substituting PID open-loop transfer function equation 35 and Ball & Beam's open-loop transfer function equation 31 into equation 36 yields

$$\begin{aligned}
 G_1(s) = \frac{x(s)}{r(s)} &= \frac{K_p (1 + \frac{1}{T_I s} + T_D s) \times \frac{c}{s^2}}{1 + K_p (1 + \frac{1}{T_I s} + T_D s) \times \frac{c}{s^2}} \\
 &= \frac{K_p (1 + \frac{1}{T_I s} + T_D s) \times c}{s^2 + K_p (1 + \frac{1}{T_I s} + T_D s) \times c} \\
 &= \frac{K_p (s + \frac{1}{T_I} + T_D s^2) \times c}{s^3 + K_p (s + \frac{1}{T_I} + T_D s^2) \times c} \\
 &= \frac{K_p T_D s^2 c + K_p s c + \frac{K_p}{T_I} c}{s^3 + K_p T_D s^2 c + K_p s c + \frac{K_p}{T_I} c} \tag{37}
 \end{aligned}$$

⇨ $K_P = K_P, K_P/T_I = K_I, K_P K_D = K_D$, Equation 37 can be derived as

$$G_1(s) = \frac{x(s)}{r(s)} = \frac{K_p s c + K_I c + K_D s^2 c}{s^3 + K_p s c + K_I c + K_D s^2 c} \tag{38}$$

According to equation 38, the response curve of the transfer function can be changed by adjusting the parameters of K_P, K_I, K_D .

3.3.2 Design of Fuzzy PID Controller

The fuzzy controller designed in this study is a two-input and three-output fuzzy PID controller. The input is the error between the given value of the small ball relative to the fixed axis end and the actual value, $e = r(k) - y(k)$, and the error change rate, $ec = e(k) - e(k-1)$. The output is the corrected value of the PID parameter $\Delta K_P, \Delta K_I, \Delta K_D$.

According to the physical model and experimental verification of the Ball & Beam system by experts, the basic domain of error e in this design is taken as $[-0.4, 0.4]$, and the basic domain of error change rate ec is taken as $[-0.71, 0.71]$. With output variables $\Delta K_P, \Delta K_I$, and K_D , their domain is taken as $[-0.3, 0.3], [-0.06, 0.06]$, and $[-3, 3]$, respectively.

In order to obtain the input of the fuzzy controller, it is necessary to fuzzify the precise quantity, that is, multiply the input quantity by the corresponding quantization factor, and convert it from the basic domain to the corresponding fuzzy domain. The quantization factor of the error is $\alpha_e = \frac{E}{e} = \frac{6}{0.4} = 15$, and the factors of the error rate of change are e and

$\alpha_{ec} = \frac{EC}{ec} = \frac{6}{0.71} = 8.5$. The control quantity obtained through the fuzzy control algorithm is a fuzzy quantity that needs

to be multiplied by the proportion factor and converted into the basic universe. The proportion factor

$\alpha_{\Delta K_P} = \alpha_{\Delta K_I} = \alpha_{\Delta K_D} = 1$ of the output variables $\Delta K_P, \Delta K_I$, and K_D is taken.

Divide the fuzzy universe of input variables and output variables ΔK_P , ΔK_I , and K_D into 7 fuzzy subsets, namely NB, NM, NS, ZO, PS, PM, and PB, representing negative large, negative medium, negative small, zero, positive small, positive medium, and positive large, respectively. The membership functions of input variables and output variables ΔK_P , ΔK_I , and K_D are all triangular (Figure 10).

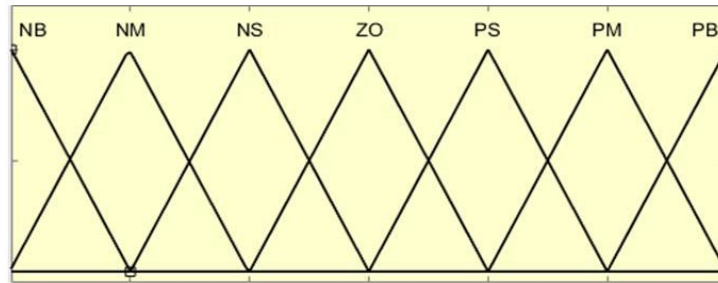


Fig.10 Membership Function of e , ec , ΔK_P , ΔK_I , and K_D

The fuzzy rule statement used by the fuzzy controller with output variables ΔK_P , ΔK_I , and K_D is as follows:

If E is α and EC is β , then U is γ . Among them, α , β , and γ represent the fuzzy sets corresponding to each variable.

Based on the impact of PID parameters on system performance and the principle of parameter tuning, the expert's experience and cognitive processing have resulted in 49 control rules, as shown in Table 2 to Table 4.

Table 2 K_P Fuzzy Rule Table

K_P \ E \ EC	NB	NM	NS	Z	PS	PM	PB
NB	PB	PB	PM	PM	PS	Z	Z
NM	PB	PB	PM	PS	PS	Z	NS
NS	PM	PM	PM	PS	Z	NS	NS
Z	PM	PM	PS	Z	NS	NM	NM
PS	PS	PS	Z	NS	NS	NM	NM
PM	PS	Z	NS	NM	NM	NM	NB
PB	Z	Z	NS	NM	NM	NB	NB

Table 3 K_I Fuzzy Rule Table

K_I \ E \ EC	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	NM	NM	NS	Z	Z
NM	NB	NB	NM	NS	NS	Z	Z
NS	NB	NM	NS	NS	Z	PS	PS
Z	NM	NM	NS	Z	PS	PM	PM

PS	NM	NS	Z	PS	PS	PM	NM
PM	Z	Z	PS	PS	PM	PB	PB
PB	Z	Z	PS	PM	PM	PB	PB

Table 4 K_D Fuzzy Rule Table

K_D	E	NB	NM	NS	Z	PS	PM	PB
	EC	NB	NM	NS	Z	PS	PM	PB
NB		PS	NS	NB	NB	NB	Z	Z
NM		PS	NS	NB	NM	NM	NS	Z
NS		Z	NS	NM	NM	NS	NS	Z
Z		Z	NS	NS	NS	NS	NS	Z
PS		Z	Z	Z	Z	Z	Z	Z
PM		PB	NS	PS	PS	PS	PS	PB
PB		PB	PM	PM	PM	PS	PS	PB

This design system uses the Mamdani inference method to perform fuzzy inference on the established fuzzy rules, thereby obtaining control variables. Using the center of gravity method to deblur the fuzzy quantities expressed in language, accurate values of ΔK_p , ΔK_i , and K_D are obtained. In addition, by multiplying the values obtained through fuzzy reasoning and deblurring by the corresponding proportional factors, the incremental adjustment values of PID parameters can be obtained. Then, by substituting equations 39, 40, and 41, the control parameters of the PID controller can be obtained.

$$K_p = K_{p0} + \Delta K_p \tag{39}$$

$$K_i = K_{i0} + \Delta K_i \tag{40}$$

$$K_D = K_{D0} + \Delta K_D \tag{41}$$

3.4 Simulink Simulation of Ball & Beam System

3.4.1 Simulation of Traditional PID Controller

This section uses the tools provided by Simulink to analyze the response curve, input signal, PID response curve, and error response curve of the system, as shown in Figure 12–15, including the position, velocity, etc. of the ball and beam. You can use such as Block, Scope and XY Graph to plot the changes in the output signal (Figure 11). Evaluate the performance of the PID controller, including indicators such as overshoot and steady-state error. Regarding the PID control scheme, the Ball & Beam system adjusts the parameters of the PID controller according to the requirements of stability, speed, and accuracy for the performance of the PID control system. In Figure 11, the PID control parameters are set as $\Delta K_p=17$, $\Delta K_i=10$, and $K_D=5$. Figure 16 shows the PID control simulation flowchart based on the Ball & Beam system. From the response curve of the system, it can be seen that the Ball & Beam system ultimately tends to a stable state with a lower overshoot, that is, the output tends to an ideal state of entry and exit, thereby achieving the function of controlling the position of the ball.

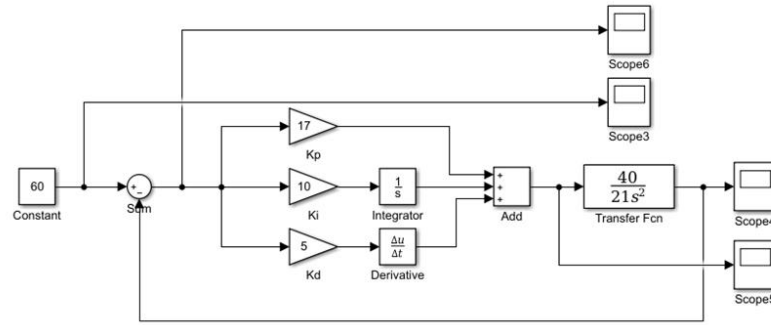


Fig 11.PID Control Principle Diagram of Ball & Beam System

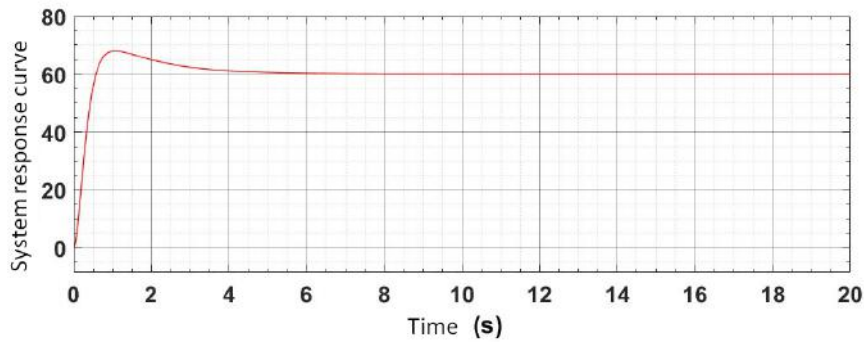


Fig 12. Response curve of Ball & Beam system when $\Delta K_P=17$, $\Delta K_I=10$, and $K_D=5$

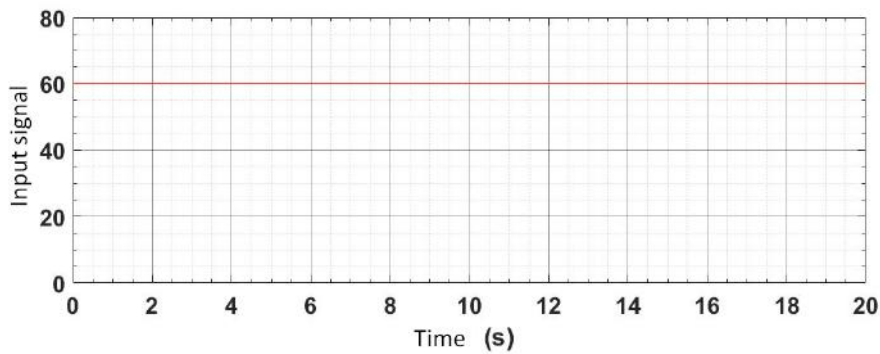


Fig.13 Input signal of Ball & Beam system when $\Delta K_P=17$, $\Delta K_I=10$, and $K_D=5$

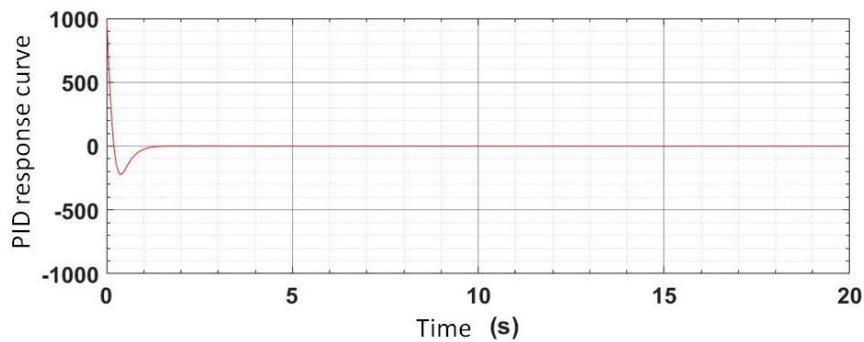


Fig.14 PID response curve of Ball & Beam system when $\Delta K_P=17$, $\Delta K_I=10$, and $K_D=5$

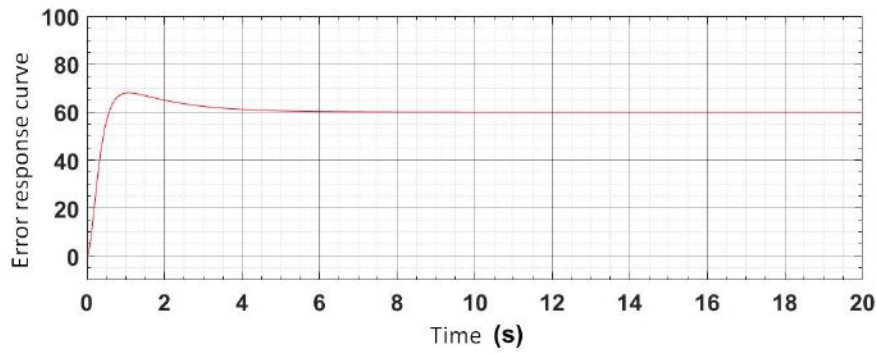


Fig.15 Error response curve of Ball & Beam system when $\Delta K_P=17$, $\Delta K_I=10$, and $K_D=5$

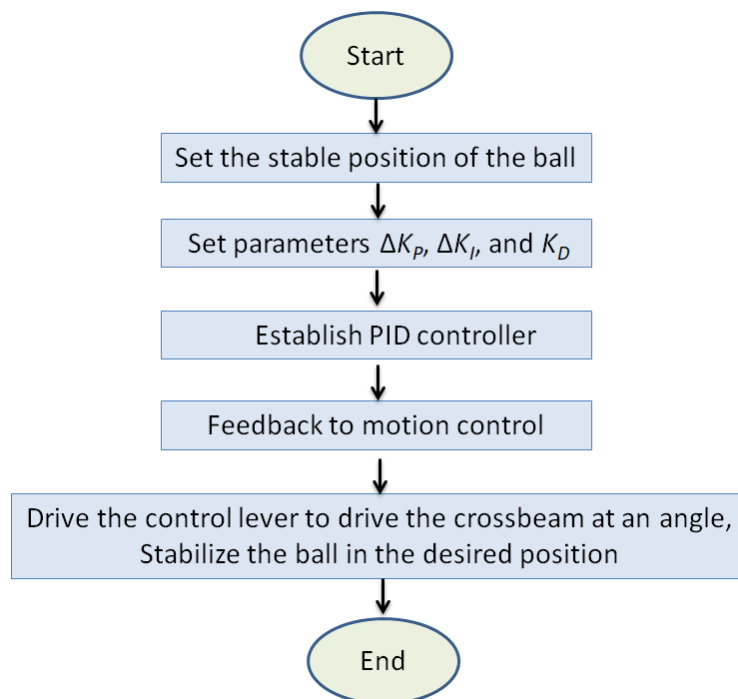


Fig.16 Ball & Beam System PID Control Simulation Flowchart

3.4.2 Fuzzy Logic System Tool Box

Before establishing a fuzzy logic controller, it is necessary to set the fuzzy logic system Tool Box parameters in Simulink. As shown in Figure 17. Figure 18 uses the parameter setting process of the membership function in Tool Box to establish the membership functions of e , ec , ΔK_P , ΔK_I , and K_D .

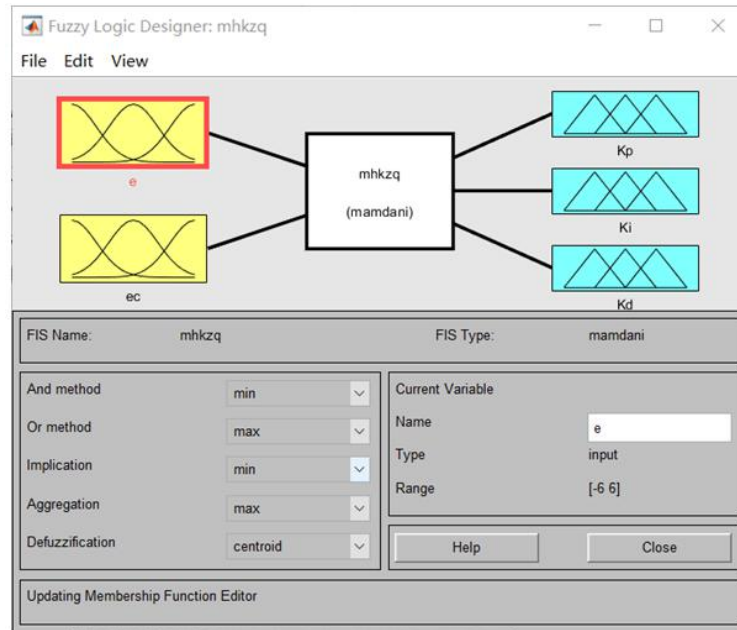


Fig.17 Fuzzy Logic System Tool Box

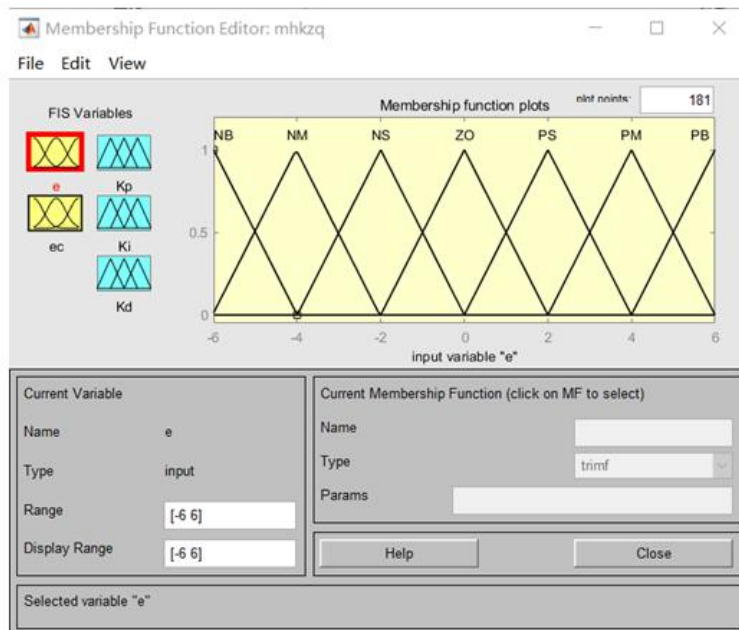


Fig.18 Membership Function of Fuzzy Logic System

After setting the above parameters, they can be added to the rule editor, and all output surfaces of the fuzzy inference system can be observed in the output surface observer. Figure 19 shows the PID control simulation flowchart based on the Ball & Beam system. Figure 20 compares the fuzzy PID controller with the traditional PID controller to verify the control effect of the fuzzy PID controller.

Through Simulink simulation with appropriate parameters, traditional PID controllers have good control effects on control objects. That can assume precise mathematical models and meet the requirements of system control accuracy and rise

time. However, there are problems, such as long adjustment times. The fuzzy PID controller can adjust the PID parameters of the system in real time. By doing so, the PID parameters can be more suitable for the control requirements of the system, resulting in better control effects than traditional PID controllers (Figure 21).

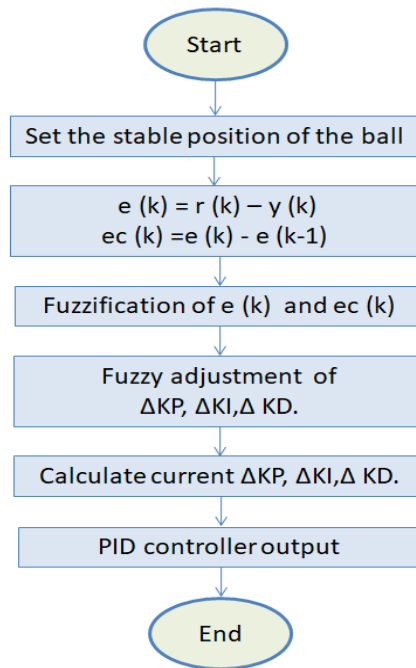


Fig.19 Workflow of Fuzzy PID Controller

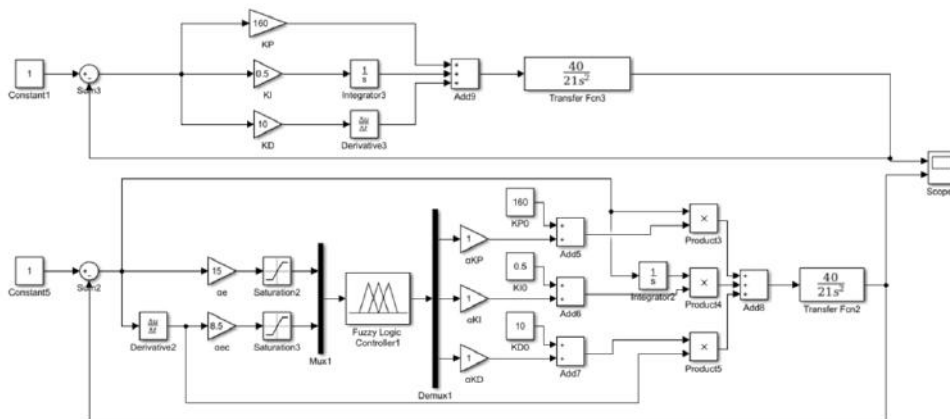


Fig.20 Simulink Simulation of Ball & Beam System's Traditional PID and Fuzzy PID Control Systems

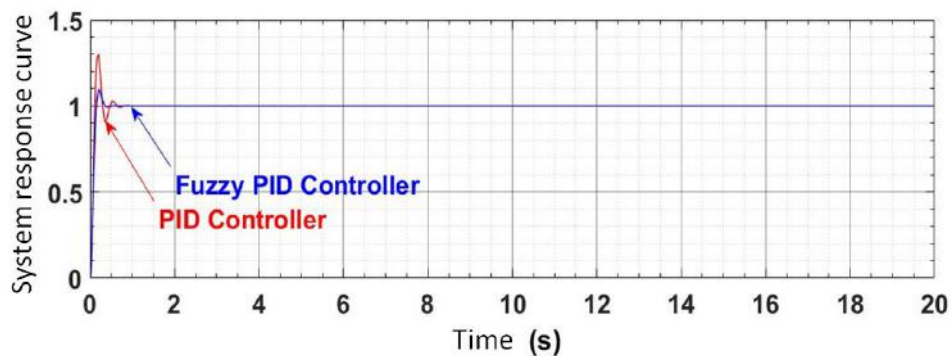


Fig.21 Comparison of Response Curves between Traditional PID and Fuzzy PID Controller Systems

IV. CONCLUSIONS

The inherent Ball & Beam system is a typical open-loop, unstable system. This article analyzes the motion of the inherent Ball & Beam system, establishes a mathematical model of the Ball & Beam system through Lagrangian equations, and designs a PID controller and a fuzzy PID controller to control the Ball & Beam system, respectively. By adjusting their proportional constant, integral constant and differential constant parameters, the Ball & Beam system can be quickly controlled. Accurately reaching a stable state and ultimately achieving a stable system. The superiority of fuzzy PID controllers over PID control was verified through Simulink simulation. Under the action of a fuzzy PID controller, the system has fast response speed, short adjustment time, and high steady-state accuracy.

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