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A Fuzzy Robust Controller for Robotic Systems

Lafi Alnufaie

Department of Electrical Engineering, Engineering College, Shaqra University, Shaqra 11961, Saudi Arabia

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***Keywords*— Type-2 Fuzzy Systems, Robust Control, Robotic system.**

***Abstract*— *In this work a combination of type 2 fuzzy logic and nonsingular fast sliding mode technique is proposed to design a robust controller for a robotic system. Indeed, a nominal type 2 fuzzy model is used to contract the equivalent control signal. The switching signal is designed using adaptive type 2 fuzzy systems to overcome the knowledge of the upper bounds of uncertainties and external disturbances. Several simulation results are given to show the efficiency of the proposed approach.*

I. INTRODUCTION

Widely used in many applications, sliding mode control can be considered a very popular approach to ensure good tracking performances against external disturbances [1]. Despite its simple design procedure and good tracking performances, sliding mode control has two major disadvantages. Thes first one is the chattering phenomena introduced by using signum function in the control signal. The second disadvantage lies in its time convergence, which cannot impose. Several improvements have been proposed in the literature to reduce the chattering phenomena. In [2], the switching signal is smoothed by using a low-pass filter. An adaptive fuzzy system has been used in [3] to substitute the switching control and, hence, to eliminate the chattering phenomenon. However, this improvement needs a tradeoff between the smoothness of the switching signal and tracking performances. Second order sliding mode control have been also presented a good solution to chattering but the design procedure is *complex* and the requires a good knowledge of the studied system [4].

Recently, terminal sliding mode control have been developed, where a nonlinear surface is used [5-6]. However, these kinds of controllers suffer from singularity problem due to presence of terms with negative fractional powers. This problem can be resolved by using a nonsingular terminal sliding mode controller [7-8]. Nevertheless, this improvement was obtained at the expense of the convergence time which becomes slower. Nonsingular fast terminal sliding mode controller have been developed to overcome singularity and to obtain fast convergence time [9]. Thus, in this paper, we propose a nonsingular fast terminal sliding mode controller for a robotic system which guarantees finite-time convergence, fast speed when the states are far from the origin, avoidance of singularity and without chattering. The control is developed using a fuzzy nominal model witch avoids using approximating system dynamics. Furthermore, adaptive type 2 fuzzy systems have been used to avoid a well-knowledge of the upper bounds of both uncertainties and external disturbances.

The remainder of this paper is organized as follows: Section 2 is dedicated to introduce type 2 fuzzy systems. In Section 3, problem statement of controlling a robotic system is treated. Section 4 is dedicated to the controller design and stability analysis. Simulation and results are given in Section 5 to show the effectiveness of the proposed approach. Finally, the conclusion is provided.

II. INTERVAL TYPE-2 FUZZY LOGIC SYSTEM

Fuzzy Logic Systems are known as the universal approximators and have several applications in control design and identification. A type-1 fuzzy system consists of four major parts: fuzzifier, rule base, inference engine, and defuzzifier. A T2FLS is very similar to a T1FLS [10] the major structure difference being that the defuzzifier block of a type-1 fuzzy system is replaced by the output processing block in a type-2 fuzzy system, which consists of type-reduction followed by defuzzification.

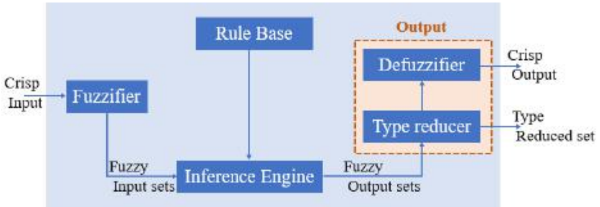

Fig. 1: Structure of a type-2 fuzzy logic system

In a interval type-2 fuzzy system, a triangular fuzzy set is defined by a lower and upper set as shown in figure 2.

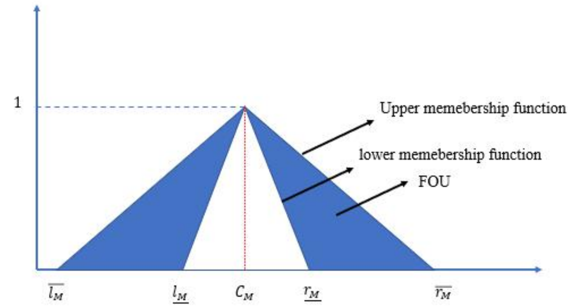

Fig. 2: Interval type-2 triangular fuzzy sets

It is clear that the interval type-2 fuzzy set is in a region bounded by an upper membership function and a lower membership function denoted as $\bar{\mu}_{\tilde{A}(x)}$ and $\underline{\mu}_{\tilde{A}}$ respectively and is named a foot of uncertainty (FOU). Assume that there are M rules in a type-2 fuzzy rule base, each of them has the following form:

$$R^i: \quad IF\ x_1\ is\ \tilde{F}_1^i\ and\ \dots\ and\ x_n\ is\ \tilde{F}_n^i\ THEN\ y\ is\ [w_l^i\ w_r^i]$$

Where x_j $j = 1,2,\dots,n$ and y are the input and output variables of variables of the type 2 fuzzy system, respectively, the $\tilde{F}_j^i$ is the type 2 fuzzy sets of antecedent part, and $[w_l^i\ w_r^i]$ is the weighting interval set in the consequent part. The operation of type-reduction is to give a type-1 set from a type-2 set. In the meantime, the firing strength F_i for the i^{th} rule can be an interval type-2 set expressed as:

$$F^i \equiv [\underline{f}^i, \overline{f}^i]$$

Where:

$$\begin{cases} \underline{f}^i = \underline{\mu}_{\tilde{F}_1^i}(x_1) * \dots * \underline{\mu}_{\tilde{F}_n^i}(x_n) \\ \overline{f}^i = \bar{\mu}_{\tilde{F}_1^i}(x_1) * \dots * \bar{\mu}_{\tilde{F}_1^i}(x_n) \end{cases}$$

In this work, the center of set type-reduction method is used to simplify the notation. Therefore, the output can be expressed as:

$$y_{cos}(x) = [y_l\ ; y_r]$$

Where $y_{cos}(x)$ is also an interval type 1 set determined by left and right most points (y_l and y_r), witch can be derived from consequent centroid set $[w_l^i\ w_r^i]$ (either $\underline{w}^i$ or $\overline{w}^i$) and the firing strength $f^i \in F^i \equiv [\underline{f}^i, \overline{f}^i]$. The interval set $[w_l^i\ w_r^i]$ $(i = 1, \dots, M)$ should be computed or set first before the computation of $y_{cos}(x)$. Hence, left most point y_l and right most point y_r can be expressed as:

$$\begin{cases} y_l = \dfrac{\sum_{i=1}^{M} f^i w_l^i}{\sum_{i=1}^{M} f^i} \\ y_r = \dfrac{\sum_{i=1}^{M} \overline{f}^i w_r^i}{\sum_{i=1}^{M} \overline{f}^i} \end{cases} \qquad (1)$$

Using the center of set type reduction method to compute y_l and y_r the defuzzified crisp output from an interval type 2 fuzzy logic system can be obtained according to the following equation:

$$y(x) = \frac{y_l + y_r}{2} \qquad (2)$$

Which can be rewritten on the following vectorial form:

$$y(x) = \Psi^T(x).w \qquad (3)$$

Where $\Psi^T(x)$ represents the regressive vector and w the consequent vector containing the conclusion values of the fuzzy rules.

III. PROBLEM STATEMENT

Let us consider the dynamic equation of n degree-of-freedom robotic manipulators as follows

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q,\dot{q}) = \Gamma(t) + \Gamma_{ext}(t) \qquad (4)$$

where q, $\dot{q}$ and $\ddot{q} \in \mathbb{R}^n$ are the vector of joint position, joint velocity, and joint acceleration, respectively. $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix of centrifugal and Coriolis forces, $G(q) \in \mathbb{R}^n$ is the vector of gravitational forces, $\Gamma(t) \in \mathbb{R}^n$ is the vector of input joint torque and $\Gamma_{ext}(t) \in \mathbb{R}^n$ is the vector of unknown external disturbances.

For practical applications, it is impossible to know the exact dynamic model of the robotic manipulators. Hence, the above dynamic quantities can be expressed as:

$$M(q) = M_0(q) + \Delta M(q)$$
$$C(q,\dot{q}) = C_0(q,\dot{q}) + \Delta C(q,\dot{q}) \quad (5)$$
$$G(q) = G_0(q) + \Delta G(q)$$

Where $M_0(q)$, $C_0(q,\dot{q})$, $G_0(q)$ are the nominal values of $M(q)$, $C(q,\dot{q})$, $G(q)$ respectively and $\Delta M(q)$, $\Delta C(q,\dot{q})$, $\Delta G(q)$ are the uncertain parts of $M(q)$, $C(q,\dot{q})$, $G(q)$ respectively.

Using equation (5), the dynamic model of the robotic manipulators can be expressed as:

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q,\dot{q}) = \Gamma(t) + \delta(q,\dot{q},\ddot{q}) \quad (6)$$

Where:

$$\delta(q,\dot{q},\ddot{q}) = \Gamma_{ext}(t) - \Delta M(q)\ddot{q} - \Delta C(q,\dot{q})\dot{q} - \Delta G(q).$$

Let define the tracking error $e = q - q_d$ and its time derivative $\dot{e} = \dot{q} - \dot{q}_d$ where q_d the desired trajectory. Then the error dynamic of the robotic manipulators with the uncertainties and disturbances can be written as:

$$\ddot{e} = f(e,\dot{e}) + g(e,\dot{e})\Gamma(t) + D(e,\dot{e}) \quad (7)$$

Where $f(e,\dot{e}) = -M_0^{-1}(q)\,[C_0(q,\dot{q})\dot{q} + G_0(q,\dot{q})] - \ddot{q}_d$, $g(e,\dot{e}) = M_0^{-1}(q)$ and $D(e,\dot{e}) = M_0^{-1}(q)\,\delta(q,\dot{q},\ddot{q})$.

As given in [14], the upper bound of lumped uncertainty can be expressed as:

$$|D(e,\dot{e})| \le a_0 + a_1|q| + a_2|\dot{q}|^2 \quad (8)$$

Where b_0, b_1 and b_2 are positive scalars.

The next task is to develop a robust controller based on nonsingular fast terminal sliding mode control allowing to tracking objectives.

IV. CONTROLLER DESIGN

To design our controller, let consider the following nonsingular terminal sliding surface:

$$S(t) = e + k_1|e|^{\alpha} sign(e) + k_2|\dot{e}|^{\beta} sign(\dot{e}) \quad (9)$$

Where k_1 and k_2 are positive constants, $1 < \beta < 2$ and $\alpha > \beta$.

The structure of this surface allows us to attain fast convergence of the tracking error to zero. Indeed, if the position initial value is far from the desired one, then the term $k_1|e|^{\alpha} sign(e)$ will be dominant, which leads to a fast convergence. In the case where the system is near the desired trajectory, the term $k_2|\dot{e}|^{\beta} sign(\dot{e})$ must ensuring a finite time convergence.

The time derivative of the sliding surface can be written as:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.\ddot{e} \quad (10)$$

Our control law will be composed from two terms. The first one, named equivalent control $\Gamma_e(t)$, is dedicated to maintain the system on the sliding surface. The second term, $\Gamma_s(t)$ called switching signal, must force the system to converge to the sliding surface. Then, to design the equivalent control law $\Gamma_e(t)$, we consider that the system is on the surface ($S(t) = 0$) and remains on ($\dot{S}(t) = 0$). In this case, the system is considered insensitive to uncertainties and external disturbances (Slotine, 1991).

Using (7) equation (10) can be rewritten as:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.[f(e,\dot{e}) + g(e,\dot{e})\Gamma_e(t)] \quad (11)$$

Then the expression of equivalent control law can be expressed as:

$$\Gamma_e(t) = -g^{-1}(e,\dot{e}).[f(e,\dot{e}) + [\beta.k_2]^{-1}|\dot{e}|^{2-\beta}(1 + \alpha.k_1|e|^{\alpha-1})\,sign(\dot{e})] \quad (12)$$

Note that, we used the fact that $\dot{e} = |\dot{e}|.sign(\dot{e})$ to writ equation (9) in a compact form.

Our next task is to determine the expression of the switching signal $\Gamma_s(t)$ allowing to force the system to reach the sliding surface in presence of uncertainties and external disturbances.

In this case, equation (10) becomes:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.[f(e,\dot{e}) + g(e,\dot{e})\Gamma(t) + D(e,\dot{e})] \quad (13)$$

Using (12), we can rewrite (10) as:

$$\dot{S}(t) = \dot{e} + \alpha.k_1|e|^{\alpha-1}\dot{e} + \beta.k_2|\dot{e}|^{\beta-1}.[f(e,\dot{e}) + g(e,\dot{e})\Gamma_e(t)] + \beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (14)$$

According to the definition of the equivalent control, equation (14) can be simplified to:

$$\dot{S}(t) = \beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (15)$$

To deduce the expression of $\Gamma_s(t)$ allowing the switching condition, we consider the following Lyapunov function:

$$V(t) = \frac{1}{2}S^2(t) \quad (16)$$

Differentiating $V(t)$ with respect to time and using (15) lead to:

$$\dot{V}(t) = S(t).\beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \quad (17)$$

Choosing $\Gamma_s(t)$ as:

$$\Gamma_s(t) = -g^{-1}(e,\dot{e})[k_{01}.S(t) + (k_{02} + a_0 + a_1|q| + a_2|\dot{q}|^2).sign(S(t))] \quad (18)$$

Where k_{01} and k_{02} are two positive scalars.

The time derivative of the Lyapunov function becomes:

$$\begin{aligned}\dot{V}(t) &= S(t).\beta.k_2|\dot{e}|^{\beta-1}.[g(e,\dot{e})\Gamma_s(t) + D(e,\dot{e})] \\ &= \beta.k_2|\dot{e}|^{\beta-1}.\left[\begin{array}{c} -k_{01}.S^2(t) \\ -\left(\begin{array}{c}k_{02} + a_0 + a_1|q| \\ +a_2|\dot{q}|^2\end{array}\right).|S(t)| \\ +D(e,\dot{e})\end{array}\right]\end{aligned} \quad (19)$$

Using the assumption (8), we obtain the following inequality:

$$\dot{V}(t) \leq \beta.k_2|\dot{e}|^{\beta-1}.[-k_{01}.S^2(t) - k_{02}.|S(t)|] \leq 0 \quad (20)$$

Based on the Lyapunov theorem, the system converges asymptotically to the sliding surface and remains on.

To prove convergence in finite time, let us take up inequality (20):

$$\dot{V}(t) \leq -\beta.k_{01}.k_2|\dot{e}|^{\beta-1}.S^2(t) - \beta.k_{02}.k_2|\dot{e}|^{\beta-1}.|S(t)| \quad (21)$$

$$\dot{V}(t) = \frac{dV(t)}{dt} \leq -\underbrace{2.\beta.k_{01}.k_2|\dot{e}|^{\beta-1}}_{\beta_1}.V(t) - \underbrace{\sqrt{2}\beta.k_{02}.k_2|\dot{e}|^{\beta-1}}_{\beta_2}.V^{\frac{1}{2}}(t) \quad (22)$$

Then we can obtain:

$$dt \leq \frac{-dV(t)}{\beta_1.V(t)+\beta_2.V^{\frac{1}{2}}(t)} = -2.\frac{dV^{\frac{1}{2}}(t)}{\beta_1.V^{\frac{1}{2}}(t)+\beta_2} \quad (23)$$

If we consider that the system converges to 0 at $t = t_r$ implies that:

$$\int_0^{t_r} dt \leq \int_{V(0)}^{V(t_r)} \frac{-2.dV^{\frac{1}{2}}(t)}{\beta_1.V^{\frac{1}{2}}(t)+\beta_2} = \left[-\frac{2}{\beta_1} ln\left(\beta_1 V^{\frac{1}{2}}(t) + \beta_2\right)\right]_{V(0)}^{V(t_r)} \quad (24)$$

Hence,

$$t_r \leq \frac{2}{\beta_1} ln\left(\frac{\beta_1 V^{\frac{1}{2}}(0)+\beta_2}{\beta_2}\right) \quad (25)$$

Consequently, the control law $\Gamma(t) = \Gamma_e(t) + \Gamma_s(t)$, whose terms are defined by equations (12) and (18), guarantees the asymptotic stability of the closed loop system and the convergence of the tracking error in a finite time.

Nevertheless, it is very difficult if not possible to know the exact values of the scalars a_0, a_1 and a_2. To overcome this problem, we propose to approximate them by three adaptive type 2 fuzzy systems $\hat{a}_0 = \Psi^T(e,\dot{e}).w_0$, $\hat{a}_1 = \Psi^T(e,\dot{e}).w_1$ and $\hat{a}_2 = \Psi^T(e,\dot{e}).w_2$. According to the universal approximation theorem, there exists an optimal values of type 2 fuzzy systems we can write:

$$\begin{aligned} a_0 &= \Psi^T(e,\dot{e}).w_0^* \\ a_1 &= \Psi^T(e,\dot{e}).w_1^* \\ a_2 &= \Psi^T(e,\dot{e}).w_2^* \end{aligned} \quad (26)$$

Where w_0^*, w_1^* and w_2^* represent the optimal values of w_0, w_1 and w_2 respectively.

Consequently, the control laws become:

$$\begin{aligned} \Gamma(t) &= \Gamma_e(t) + \Gamma_s(t) \\ \Gamma_e(t) &= -g^{-1}(e,\dot{e}).\left[f(e,\dot{e}) + [\beta.k_2]^{-1}|\dot{e}|^{2-\beta}(1+\alpha.k_1|e|^{\alpha-1})\,sign(\dot{e})\right] \\ \Gamma_s(t) &= -g^{-1}(e,\dot{e})[k_{01}.S(t) + (k_{02} + \hat{a}_0 + \hat{a}_1|q| + \hat{a}_2|\dot{q}|^2).sign(S(t))] \end{aligned} \quad (27)$$

These modified control laws allow to ensure the convergence to the reference trajectory in a finite time.

To deduce the adaptation laws of the three adaptive fuzzy system, we consider the new Lyapunov function:

$$V(t) = \frac{1}{2}S^2(t) + \beta.k_2\left(\frac{1}{2\gamma_0}(w_0 - w_0^*)^2 + \frac{1}{2\gamma_1}(w_1 - w_1^*)^2 + \frac{1}{2\gamma_2}(w_2 - w_2^*)^2\right) \quad (28)$$

Using the control laws (27) and the following adaptation laws:

$$\begin{aligned} \dot{w}_0 &= \gamma_0\Psi^T(e,\dot{e}).|S(t)|.|\dot{e}|^{\beta-1} \\ \dot{w}_1 &= \gamma_1\Psi^T(e,\dot{e}).|S(t)|.|\dot{e}|^{\beta-1}|e| \\ \dot{w}_2 &= \gamma_0\Psi^T(e,\dot{e}).|S(t)|.|\dot{e}|^{\beta} \end{aligned} \quad (29)$$

And following the same mathematical development used previously, the time derivative of the Lyapunov function (28) becomes:

$$\dot{V}(t) \leq \beta.k_2|\dot{e}|^{\beta-1}.[-k_{01}.S^2(t) - k_{02}.|S(t)|] \leq 0 \quad (30)$$

Thus, the convergence of the closed loop system to the reference trajectory in a finite time is guaranteed.

V. SIMULATION AND RESULTS

To show the performances of the performances of the proposed approach, we consider a one link robot, shown in fig. 3, whose dynamics equation is given by:

$$ml^2\ddot{q} + mglcos(q)\dot{q} + mglsin(q) = \Gamma + \Gamma_{ext}(t)$$

With: $m_1 = 1Kg; l_1 = 1m; g = 9.8ms^{-2}$

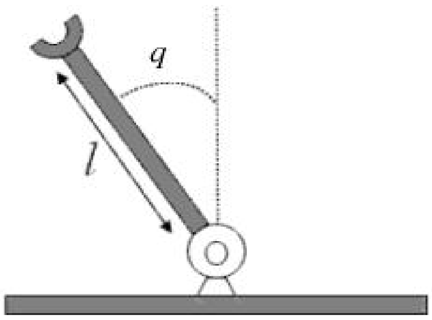

Fig. 3: One link robot manipulator

To construct the type 2 fuzzy nominal model, we consider that the position q is constrained within $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$, which leads to 3 fuzzy rules. Each one of them gives the relation between the equilibrium point and the corresponding local model. Then, each rule uses a type 2 fuzzy sets in the antecedent part to describe the equilibrium point and the consequent part the corresponding local model. Using the product as an interference engine, the method of center set for the reduction type and center of gravity for defuzzification, the output fuzzy system will be giving the type 2 fuzzy nominal model.

Fig. 4 gives the angular position and velocity for two initial positions. The convergence to zero in a finite time is well shown in fig. 5. To illustrate the efficiency of the proposed approach we have used a more complex trajectory,

$q_q = 0.5\cos((t) + 0.5\sin(2t)$. Fig. 6 gives the angular position and velocity for two initial values. These results show also the good tracking performances and the convergence to the reference trajectory in a finite time. Furthermore, the control signal given by fig. 7 the elimination of the chattering phenomenon and the smooth control signal. We can conclude that the proposed approach ensures high tracking precision, fast response, singularity avoidance and strong robustness to external disturbances and modeling uncertainties.

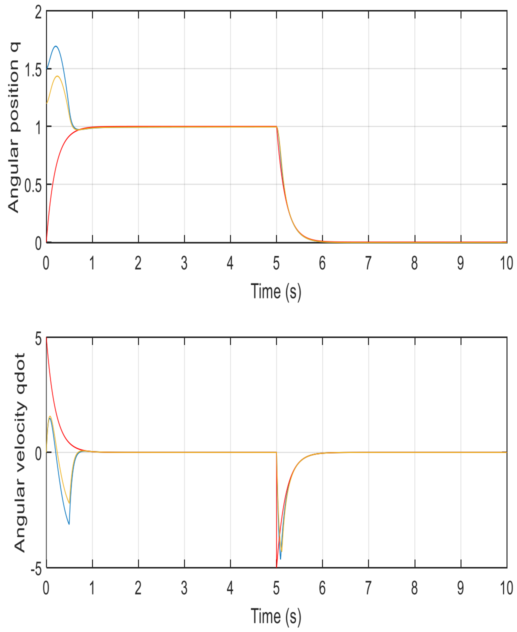

Fig. 4: Angular position and velocity tracking

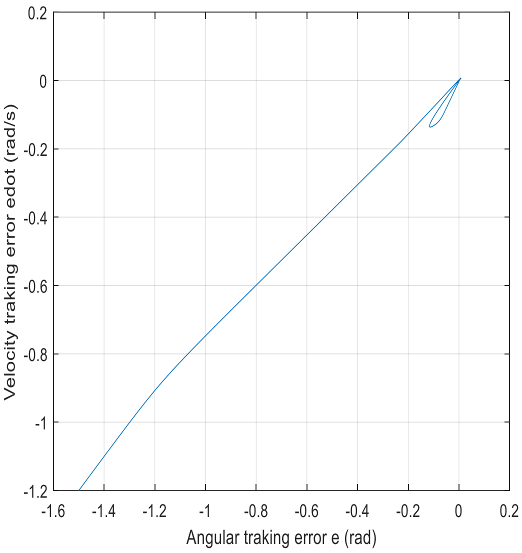

Fig. 5: Error phase plane

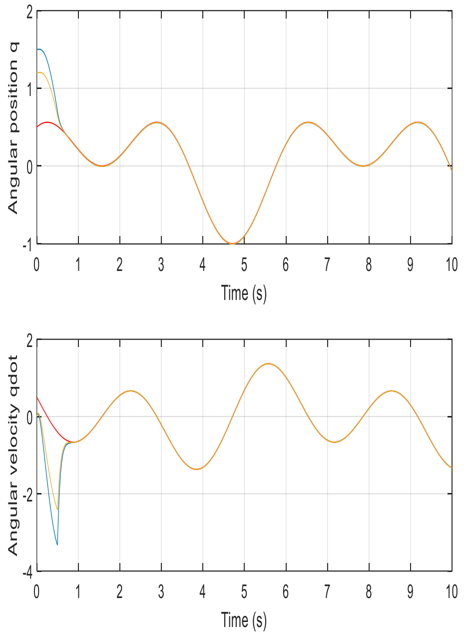

Fig. 6: Angular position et velocity tracking

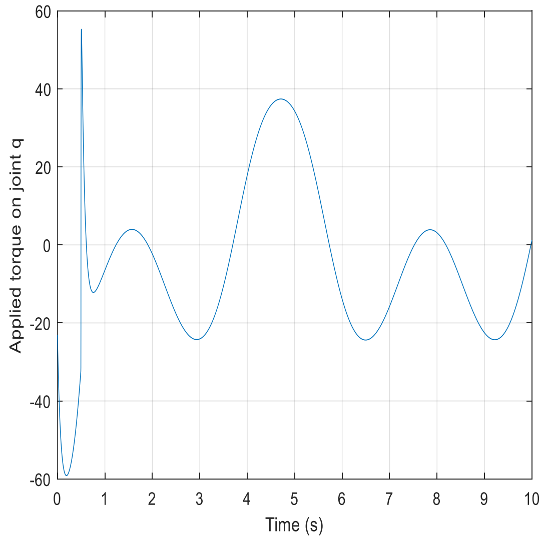

Fig. 7: Applied control signal

VI. CONCLUSION

In this paper, a combination of type 2 fuzzy logic and nonsingular fast sliding mode control for robotic systems is presented. The proposed approach allows to overcome the drawbacks encountered in classical cases, thanks to type 2 fuzzy logic. Several simulation results have presented to show the efficiency of the proposed approach despite the presence of modeling uncertainties and external disturbances. As perspective of this work to improve the control law design by reducing number of used parameters.

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