International Journal of Advanced Engineering Research and Science (IJAERS)
Peer-Reviewed Journal
ISSN: 2349-6495(P) | 2456-1908(O)
Vol-10, Issue-1; Jan, 2023
Journal Home Page Available: https://ijaers.com/
Article DOI: https://dx.doi.org/10.22161/ijaers.101.11

Breaker Indices in Water Wave Formulated from *Kinematic Free Surface Boundary Condition, Conservation Equation of Wave Number* and *Equation of Energy Conservation*

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Received: 07 Dec 2022,

Receive in revised form: 03 Jan 2023,

Accepted: 12 Jan 2023,

Available online: 25 Jan 2023

***Keywords*— Breaking parameters, breaker height, breaker depth and breaker length.**

***Abstract*— *In this research, the breaker index equations are formulated using the kinematic free surface boundary condition. By substituting the potential velocity equation for the solution of Laplace's equation in this equation, it is obtained the wave amplitude function equation. From the wave amplitude function equation two breaker indices are extracted, they are the breaker length index which is the ratio between the breaker height and the breaker length; and the breaker depth index which is the ratio between the breaker height and the breaker depth. The next breaker index, which is a ratio between breaker depth and breaker length, is obtained from the wave number conservation law. Consistency testing of the three breaker index equations obtained shows that there is consistency in the three equations. Consistency testing is done by using the connectivity equation, where a breaker index is the product of the multiplication of the other two breaker indexes. The breaker height index, which is the ratio between the breaker height and the deep water wave height, is obtained by substituting the breaker length index in the energy conservation equation. Thus the breaker height equation is obtained which works a function of the breaker length at the breaking point. With the availability of the four breaker indexes, the breaking parameter can be calculated easily.*

I. INTRODUCTION

The breaking parameter is the wave characteristics at the time of breaking. There are three breaking parameters, namely the *breaking wave height* or also known as the breaker height, the *water depth at the breaking point* which is called the breaker depth and the *wavelength at the breaking point* which is called the breaker length. Breaker index is a ratio between breaking parameters. The breaker depth index is the ratio between the breaker height and the breaker depth, the breaker length index is the ratio between the breaker height and the breaker length. There is a breaker index which is rarely researched or used, namely the ratio between breaker depth and breaker length; this breaker index is hereinafter referred to as the breaker depth-length index. Breaker height index is the ratio between breaking wave height and deep water wave height.

Several explicit equations to the breaker depth index were put forward by, among others, McCowan (1894), Weggel (1972), Galvin (1969), Collins and Weir (1969), Madsen (1976) and Smith and Krauss (1989).

There is a critical wave steepness criterion, which is the *wave steepness before breaking* which is the ratio between the wave height and wavelength before breaking. This criterion can also be considered as a *breaker length index*. Two researchers who put forward the criterion of critical wave steepness are Michell (1894) and Toffoli et al (2010). The breaker length index is in the form of an implicit equation which is a function of another breaker index, namely the *Mieche equation* (1944). This equation connects the breaker length index with the breaker depth-length index. Many researchers have developed the breaker length index by modifying the Mieche equation (1944), including Battjes and Jansen (1978), Ostendorf and Madsen (1979) and Rattanapittikon and Shibayama (2000). The various breaker index equations are formulated separately, where the breaker depth index is formulated only by examining the breaker depth index, the breaker length index is formulated only by examining the breaker length index, as well as the breaker height. There should be a relationship or connectivity between breaker indexes, where the value of one breaker index is related to the value of another breaker index. In the breaker index equations obtained in this study, an examination of the linkages between breaker indexes was carried out using the connectivity equation. As a result, it is found that there is connectivity between breaker indexes.

II. THE VELOCITY POTENTIAL EQUATION

The complete velocity potential solution of the Laplace equation (Dean (1991)) using the variable separation method is:

$$\phi(x,z,t) = G\,(\cos kx + \sin kx)\cosh k(h+z)\sin \sigma t \quad (1)$$

Where x is the horizontal axis, z is vertical axis and t is time, while h is water depth. G, k dan σ is called the wave constant, G is the energy transmission rate, k is wave number, $k = \frac{2\pi}{L}$ where L is a wavelength and $\sigma = \frac{2\pi}{T}$ is angular frequency while T is for wave period.

The three wave constants need to be determined by their value or equation. As shown in (1), the velocity potential consists of two components, namely the cos and the sin component. In both components there is a point where $\cos kx = \sin kx$, these points are called characteristic points. Analysis of wave constants will be easier if done at characteristic points, where the constants obtained will satisfy both wave components. By using only components $\cos kx$ then the velocity potential equation becomes,

$$\phi(x,z,t) = G\,\cos kx \cosh k(h+z)\sin \sigma t \quad \ldots(2)$$

Note that the G in (2) has a double value.

III. THE WAVE AMPLITUDE FUNCTION EQUATION

The *wave amplitude function* is the relationship between wave amplitude and other wave constants. This equation is obtained by substituting the velocity potential for the kinematic free surface boundary condition and the equation obtained is integrated with time t to obtain the elevation water surface equation.

As a kinematic free surface boundary condition, a weighted kinematic free surface boundary condition from Hutahaean (2022) is used in the form,

$$\gamma_2 \frac{\partial \eta}{\partial t} = w_\eta - u_\eta \frac{\partial \eta}{\partial x} \quad \ldots(3)$$

γ_2 is a coefficient that is greater than 1, but the value of this coefficient has no effect on the breaker index. $\eta(x,t)$ is the water surface elevation relative to the still water level, w_η surface vertical water particle velocity u_η surface horizontal water particle velocity.

By using the velocity potential equation,

$$u(x,z,t) = -\frac{\partial \phi}{\partial x} = Gk \sin kx \cosh k(h+z)\sin \sigma t$$

$$u_\eta = Gk \sin kx \cosh k(h+\eta)\sin \sigma t$$

$$w(x,z,t) = -\frac{\partial \phi}{\partial z} = -Gk \cos kx \sinh k(h+z)\sin \sigma t$$

$$w_\eta = -Gk \cos kx \sinh k(h+\eta)\sin \sigma t$$

Substitute these velocity equations into (3) and work on the characteristic points,

$$\frac{\partial \eta}{\partial t} = -\frac{Gk}{\gamma_2}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)\cos kx \sin \sigma t \quad \ldots\ldots(4)$$

As a periodic function, then

$$Gk\ \cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right) = constant$$

Then integration (4) can be completed by integrating the elements of $\sin \sigma t$, and is obtained water surface elevation equation of,

$$\eta(x,t) = \frac{Gk}{\sigma\gamma_2}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)\cos kx \cos \sigma t$$

Wave amplitude is defined as,

$$A = \frac{Gk}{\sigma\gamma_2}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)$$

Considering that G has a double value, then

$$A = \frac{Gk}{2\sigma\gamma_2}\cosh k(h+\eta)\left(\tanh k(h+\eta) + \frac{\partial \eta}{\partial x}\right)$$

A is the wave amplitude, the water surface elevation equation becomes,

$$\eta(x,t) = A \cos kx \sin \sigma t$$

At a characteristic point of space and time, $\eta = \frac{A}{2}$ dan $\frac{\partial \eta}{\partial x} = -\frac{kA}{2}$, wave amplitude function becomes

$$A = \frac{Gk}{2\sigma\gamma_2} \cosh k\left(h + \frac{A}{2}\right)\left(\tanh k\left(h + \frac{A}{2}\right) - \frac{kA}{2}\right) \qquad \ldots\ldots(5)$$

In the wave amplitude function there is an element of $\tanh k\left(h + \frac{A}{2}\right)$ which has a constant value. The constant value of the function of $\tanh k\left(h + \frac{A}{2}\right)$ is reached at a large value of h, at $h = h_0$, h_0 is the deep water depth where the influence of waves still reaches the bottom of the waters even though it is very small or close to zero. The constant value is,

$$\tanh k_0\left(h_0 + \frac{A_0}{2}\right) = 1$$

Therefore, in deep waters even at depths greater than h_0, the waves seem to move on water depth h_0, where

$$k_0\left(h_0 + \frac{A_0}{2}\right) = \theta\pi$$

θ is the deep water coefficient whose value needs to be determined. However, based on the wave number conservation law discussed in section (6), this equation applies at all depths, so it can be written in general as:

$$k\left(h + \frac{A}{2}\right) = \theta\pi \qquad \ldots\ldots(6)$$

The determination of the value θ is to use $\tanh \theta\pi = 1$. However, this criterion is very dependent on the level of accuracy used, as shown in Table (1).

Table (1) Value $\tanh \theta\pi$

θ	$\tanh \theta\pi$		
	(5)	(6)	(7)
1.95	0.99999	0.999990	0.9999905
2	0.99999	0.999993	0.9999930
2.05	0.99999	0.999995	0.9999949
2.1	1	0.999996	0.9999963
2.4	1	0.999999	0.9999994
2.45	1	1.000000	0.9999996
2.75	1	1.000000	0.9999999
2.8	1	1.000000	1.0000000

In Table (1), (5), (6) and (7) is the number of digits after the decimal point.

- With an accuracy of 5 decimal places, it is found that $\tanh \theta\pi = 1$ at $\theta = 2.1$
- With an accuracy of 6 decimal places, it is found that $\tanh \theta\pi = 1$ at $\theta = 2.45$
- With an accuracy of 7 decimal places, it is found that $\tanh \theta\pi = 1$ at $\theta = 2.80$

Due to this condition, the value of θ cannot be determined only by using the criterion of $\tanh \theta\pi = 1$, thus additional criteria are needed. The more precise determination of value θ will be done in section 9.

IV. Equation of Breaker Length Index $\frac{H_b}{L_b}$.

In (5) there is a breaking characteristic, when

$$\tanh k\left(h + \frac{A}{2}\right) - \frac{kA}{2} = 0 \qquad \ldots\ldots(7)$$

In this condition the wave amplitude function is zero. In (7) substituted (6) in the 1st term as well $k = \frac{2\pi}{L}$ dan $A = \frac{H}{2}$ in the second term it is obtained, breaker length index of

$$\frac{H_b}{L_b} = \frac{2\tanh \theta\pi}{\pi} \qquad \ldots..(8)$$

V. Equation of Breaker Depth Index $\frac{H_b}{h_b}$.

It is defined as,

$$\tanh k\left(h + \frac{A}{2}\right) = \beta k\left(h + \frac{A}{2}\right)$$

An equation is obtained as,

$$\beta - (1 - \beta)\frac{H}{4h} = 0$$

By using (6) thus,

$$\beta = \frac{\tanh \theta\pi}{\theta\pi}$$

With this equation, the breaking equation becomes,

$$\frac{H_b}{h_b} = \frac{4\tanh \theta\pi}{\theta\pi - \tanh \theta\pi} \qquad \ldots\ldots(9)$$

VI. Equation for $\frac{h_b}{L_b}$

The equation for $\frac{h_b}{L_b}$ is formulated by using the conservation equation of the wave number.

In solving the Laplace equation with the *variable separation method*, the velocity potential is considered to be a multiplication of 3 functions (Dean (1991)), that is:

$$\phi(x, z, t) = X(x)Z(z)T(t)$$

where $X(x)$ is only the function of x, $Z(z)$ is only the function of z and $T(t)$ is only for the t time. In (1), which is $Z(z)$:

$$Z(z) = \cosh k(h + z)$$

where h is for *water depth*. As the function is only of z then at sloping bottom where water depth $h = h(x)$, also for $k = k(x)$,

$$\frac{\partial Z(z)}{\partial x} = \sinh k(h + z)\frac{\partial k(h + z)}{\partial x} = 0$$

In this equation the value is zero:

$$\frac{\partial k(h + z)}{\partial x} = 0$$

If this equation is worked on $z = \frac{A}{2}$ thus,

$$\frac{\partial k\left(h + \frac{A}{2}\right)}{\partial x} = 0$$

Thus,

$$k\left(h+\frac{A}{2}\right)=constant$$

By using (6), the equation for the conservation of the waave number is obtained,

$$k\left(h+\frac{A}{2}\right)=\theta\pi$$

The equation for the conservation of the wave number applies to all water depths, including the breaker depth, therefore it applies to the breaking point,

$$k_b\left(h_b+\frac{A_b}{2}\right)=\theta\pi$$

Substitute $k_b=\frac{2\pi}{L_b}$ and $A_b=\frac{H_b}{2}$,

$$\frac{h_b}{L_b}=\frac{\theta}{2}-\frac{H_b}{4L_b}$$

Substitute $\frac{H_b}{L_b}$ with (8),

$$\frac{h_b}{L_b}=\frac{\theta}{2}-\frac{\tanh\theta\pi}{2\pi} \qquad \text{.....(10)}$$

VII. CONSISTENCY CHECKING $\frac{H_b}{h_b}$, $\frac{H_b}{L_b}$ DAN $\frac{h_b}{L_b}$

The consistency test is proof that a breaker index is the product of the multiplication of the other two breaker indexes. The consistency test is done using the connectivity equation,

$$\frac{H_b}{L_b}=\frac{H_b}{h_b}\frac{h_b}{L_b} \qquad \text{......(11)}$$

This equation states that the multiplication between the equations $\frac{H_b}{h_b}$ and $\frac{h_b}{L_b}$ must create an equation of $\frac{H_b}{L_b}$ which is the same as (8).

Substitute $\frac{H_b}{h_b}$ with (9) and $\frac{h_b}{L_b}$ with (10),

$$\frac{H_b}{L_b}==\left(\frac{4\tanh\theta\pi}{\theta\pi-\tanh\theta\pi}\right)\left(\frac{\theta}{2}-\frac{\tanh\theta\pi}{2\pi}\right)$$

It is obtained,

$$\frac{H_b}{L_b}=\frac{2\tanh\theta\pi}{\pi}$$

The equation is the same as (8). Therefore, the equations of $\frac{H_b}{h_b}$, $\frac{H_b}{L_b}$ and $\frac{h_b}{L_b}$ meet the consistency requirements. This consistency character also shows that the value of one breaker index is determined by another breaker index, or in other words there is interdependence between breaker indexes. Therefore, the formulation of the three breaker indexes should be done simultaneously so that consistency can be checked and there is connectivity between the breaker indexes.

VIII. Breaker Height Index $\frac{H_b}{H_0}$.

The wave energy at one wavelength for a sinusoidal wave is

$$E=c_E\rho gH^2L\ \text{(m)}$$

c_E is a coefficient, which in linear wave theory $c_E=\frac{1}{8}$ (Dean (1991)). ρ is water mass density, g is gravitational force, H is wave height and L is wavelength.

Based on the law of conservation of energy, the wave energy at the breaker point is the same as the wave energy in deep water,

$$c_E\rho gH_b^2L_b=c_E\rho gH_0^2L_0$$

The same elements cancel each other out,

$$H_b^2L_b=H_0^2L_0$$

Substituting the breaker length index (8), we get the breaker height equation associated with the breaker length index value

$$H_b^3=\frac{2\tanh\theta\pi}{\pi}H_0^2L_0 \qquad \text{.....(12)}$$

Hutahaean (2022) obtained the deep water wave number is,

$$k_0=\frac{\gamma_3}{\left(\gamma_2+\frac{\gamma_3}{2}\right)\frac{H_0}{2}}$$

γ_2 and γ_3 are coeficients where $\gamma_2=1.4$ and $\gamma_3=1.8$. By this wave number, the deep water wavelength is:

$$L_0=\frac{\pi\left(\gamma_2+\frac{\gamma_3}{2}\right)H_0}{\gamma_3}$$

Substituting L_0 to (12), it is obtained

$$\frac{H_b}{H_0}=\left(\frac{2\tanh(\theta\pi)\left(\gamma_2+\frac{\gamma_3}{2}\right)}{\gamma_3}\right)^{1/3} \qquad \text{....(13)}$$

IX. DETERMINATION OF VALUE OF θ

The calculation results for $\frac{H_b}{h_b}$, $\frac{H_b}{L_b}$, $\frac{h_b}{L_b}$ and $\frac{H_b}{H_0}$ for some deep water depth coefficient values θ is presented in Table (2) below. To save space, only calculation results are presented where there is a match between $\frac{H_b}{h_b}$ with previous research.

Table (2) the value θ which results $\frac{H_b}{h_b}$ that is in a line with previous research

θ	$\frac{H_b}{L_b}$	$\frac{H_b}{h_b}$	$\frac{h_b}{L_b}$	$\frac{H_b}{H_0}$
1.95	0.637	0.78	0.816	1.367
2.1	0.637	0.715	0.891	1.367
2.6	0.637	0.56	1.141	1.367

There is a match between the values of $\frac{H_b}{h_b}$ from several previous studies, where the match can be found in the value of θ that varies.

- In $\theta=1.95$, it is obtained $\frac{H_b}{h_b}=0.78$, which is in a line with:

a. Mc Cowan (1894) : $\frac{H_b}{h_b}=0.78$

b. Weggel (1972) :

$$\frac{H_b}{h_b} = \frac{gT^{21.56}/[1+exp(-19.5m)]}{gT^2 + h_b 43.75[1-exp(-19m)]}$$

m is bottom slope, for $m = 0$; $\frac{H_b}{h_b} = 0.78$

- In $\theta = 2.10$, it is obtained $\frac{H_b}{h_b} = 0.715$, which is in a line with:

a. Galvin (1969) : $\frac{H_b}{h_b} = \frac{1}{1.4-6.85\,m}$; for bottom slope $m \leq 0.07$

In $m = 0$, $\frac{H_b}{h_b} = \frac{1}{1.4} = 0.714$

b. Collins and Weir (1969) : $\frac{H_b}{h_b} = 0.72 + 5.6\,m$

In $m = 0$, $\frac{H_b}{h_b} = 0.72$

c. Madsen (1976) : $\frac{H_b}{h_b} = 0.72(1 + 6.4m)$

In $m = 0$, $\frac{H_b}{h_b} = 0.72$

- In $\theta = 2.60$, it is obtained $\frac{H_b}{h_b} = 0.56$, that matches with Smith and Krauss (1989)

$$\frac{H_b}{h_b} = \frac{1.12}{1+exp(-60m)} - 5(1 - \exp(-43m))\frac{H_0}{L_0}$$

In $m = 0$, $\frac{H_b}{h_b} = 0.56$

To determine the θ the value of $\frac{h_b}{L_b}$ is examined where $\frac{h_b}{L_b} < 1$ shows that the waterdepth is smaller than wavelength. This condition causes water depth unable to support wave hydrodynamics resulting in breaking. In $\theta = 2.6$, the value of $\frac{h_b}{L_b} > 1$, it can be assumed that breaking will not occur in that condition.

Then, to select between $\theta = 2.1$ and $\theta = 1.95$ where both result in $\frac{h_b}{L_b} < 1$, a review will be carried out based on deep water depth criteria by using the wave number conservation equation in deep waters. Equation (6) is done in deep water,

$$k_0\left(h_0 + \frac{A_0}{2}\right) = \theta\pi$$

Substitute $k = \frac{2\pi}{L}$, and the equation can be expressed as

$$\frac{h_0}{L_0} = \frac{\theta}{2} - \frac{H_0}{4L_0}$$

h_0 in this case is the transitional depth between shallow water and deep water, where the water depth is smaller than h_0 is shallow water while the water depth is greater than h_0 belongs to deep water. At the depth of the transition, $\frac{h_0}{L_0}$ should be greater than one or at least 1. This suggests that θ must be greater than 2. From this, it can be concluded that the breaker index that corresponds to Table (2) is the breaker index formulated by $\theta = 2.1$.

Breaker depth index in $\theta = 2.1$ produces a breaker depth index that is in accordance with the results of research from Galvin (1969), Collins and Weir (1969) and Madsen (1976). In addition, the values in Table (1) show that $\tanh\theta\pi$ untuk $\theta = 2.1$ has reached 1 in the accuracy of 5 decimal places while in $\theta = 1.95$ it has not reached 1. Both of these strengthen the conclusion that the appropriate value of the depth coefficient θ is 2.1, with the breaker index on Table (2).

From the above discussion it can be seen that the breaker index is determined by the deep water coefficient θ which means that the breaker index is set by determining the deep water depth. Thus there is a possibility that the diversity of breaker index obtained from previous research is also caused by differences in determining deep water depth.

It has been shown that there is a match between the breaker depth index obtained with the breaker depth index from previous studies. In the following section, the value of the breaker length index will be studied $\frac{H_b}{L_b}$ where the value of $\frac{H_b}{L_b}$ obtained is a constant for all θ that is $\frac{H_b}{L_b} = 0.637$. This value is much larger than the critical wave steepness:

a. Michell, J.H. (1894) : $\frac{H}{L} = 0.142$

b. Toffoli et al (2010) : $\frac{H}{L} = 0.170$

Then it is studied the critical wave steepness from Michell, J.H. and Toffoli et al for the associated breaker depth index values. By specifying the value of $\frac{H_b}{L_b}$ then θ can be calculate with (8), with the value of θ obtained $\frac{H_b}{h_b}$ using (9). The result for $\frac{H_b}{L_b} = 0.142 - 0.170$ is shown in Table (3).

Table (3) the value θ in some values of $\frac{H_b}{L_b} = 0.142 - 0.170$

$\frac{H_b}{L_b}$	θ	$\frac{H_b}{h_b}$	$\frac{\tanh\theta\pi}{\theta\pi}$
0.142	0.072	233.951	0.98319
0.146	0.074	220.914	0.98222
0.150	0.076	208.905	0.98121
0.154	0.079	197.819	0.98018
0.158	0.081	187.565	0.97912
0.162	0.083	178.060	0.97803
0.166	0.085	169.234	0.97691
0.170	0.087	161.023	0.97576

In $\frac{H_b}{L_b}$ from 0.142 to 0.170, it is obtained the value of θ which is smal, as much as 0.072-0.087, with $\frac{\tanh\theta\pi}{\theta\pi}$ ranges from 0.98319 to 0.97576 closer to 1. The value of $\frac{H_b}{h_b}$ obtained is very large not found in the results of previous studies. The large value is due to $\tanh\theta\pi \approx \theta\pi$.

According to (6), in small amplitude it applies $\theta\pi = kh$ thus $\frac{\tanh kh}{kh} \approx 1$, this is a shallow water long wave condition. From this condition it can be concluded that both Michell's (1894) and Toffoli et al's (2010) criteria are for long waves in shallow waters.

a. Miche's Equation Study.

Miche (1944), create a breaker index equation connecting the two breaker indexes, that is

$$\frac{H_b}{L_b} = 0.142\tanh\left(\frac{2\pi h_b}{L_b}\right) \quad \text{.......(14)}$$

The equation connects two breaker indexes of $\frac{H_b}{L_b}$ and $\frac{h_b}{L_b}$. By determining the value of $\frac{H_b}{L_b}$ thus the value of $\frac{h_b}{L_b}$ can be calculated. After that, by using the connectivity equation, $\frac{H_b}{h_b}$ can be calculated. The calculation results is shown in Table (4).

Table (4) Calculation of the breaker index using the Miche equation.

$\frac{H_b}{L_b}$	$\frac{h_b}{L_b}$	$\frac{H_b}{h_b}$
0.082	0.10483	0.78224
0.099	0.13716	0.72179
0.126	0.22428	0.5618
0,142	1.15351	0.12397

In Table (4) it can be seen that there is a match between $\frac{H_b}{h_b}$ and previous studies and in $\frac{H_b}{L_b}$ maximum which is 0.142. It was obtained that the match is in the values of $\frac{H_b}{L_b}$ and $\frac{h_b}{L_b}$ is very small, in which this is the condition of long wave in shallow water.

Then, the Miche's equation is modified to,

$$\frac{H_b}{L_b} = 0.637\tanh\left(\frac{2\pi h_b}{L_b}\right) \quad \text{.....(15)}$$

The calculation results where there are matches between $\frac{H_b}{h_b}$ and previous studies are presented in Table (5).

Table (5) Calculation of breaker index using (15).

$\frac{H_b}{L_b}$	$\frac{h_b}{L_b}$	$\frac{H_b}{h_b}$
0.636955	0.815732	0.780839
0.636980	0.880247	0.723638
0.636995	0.990467	0.643126
0.637000	1.153513	0.552226

Table (5) indicates that there is conformity with Table (2) in the value of $\frac{H_b}{L_b}$. $\frac{h_b}{L_b}$ and also $\frac{H_b}{h_b}$. Then Miche's equation can be modified into an equation for short waves by using $\frac{H_b}{L_b} \approx 0.637$ or in other words, $\frac{H_b}{L_b} \approx 0.637$ is according to Miche's equation.

X. CALCULATION RESULTS OF BREAKING PARAMETERS USING THE BREAKER INDEX

In section 9, it was obtained $\frac{H_b}{L_b} = 0.637$, $\frac{H_b}{h_b} = 0.715$, $\frac{h_b}{L_b} = 0.891$ and $\frac{H_b}{H_0} = 1.367$ in $\theta = 2.1$. An example of calculating the breaking parameter with the breaker indexes for several deep water wave heights H_0 is presented in Table (6).

Table (6) The calculation of parameters breaking

H_0 (m)	T (sec)	H_b (m)	L_b (m)	h_b (m)
1	4.61	1.37	2.15	1.91
1.5	5.65	2.05	3.22	2.87
2	6.53	2.73	4.3	3.83
2.5	7.3	3.42	5.37	4.78
3	7.99	4.1	6.44	5.74

Wave period in Table (6) is obtained from the relation of wave period with wave height (Hutahaean (2022)),

$$T = \sqrt{\frac{8\pi^2\left(\gamma_2+\frac{\gamma_3}{2}\right)^2\frac{H_0}{2}}{g}} \quad \text{.....(16)}$$

To get an overview of the conditions resulting from the calculation of the breaker parameter with the breaker index equation obtained, we will review the breaker height from the previous breaker height equations.

The empirical breaker height index equation $\frac{H_b}{H_0}$ is quite a lot and can be divided into two groups, namely those that use the bottom slope as a parameter and those that do not use. In this study, the breaker depth index is used without a bottom slope as a parameter and or the bottom slope is given a value of zero. Researchers in this group include Komar and Gaughan (1972), Larson and Krauss (1989), Smith and Krauss (1990) and Gourlay (1992).

Komar and Gaughan (1972) :

$$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-0.2} \quad \text{......(17)}$$

Larson and Kraus (1989 :

$$\frac{H_b}{H_0} = 0.53\left(\frac{H_0}{L_0}\right)^{-0.24} \quad \text{......(18)}$$

Smith and Kraus (1990) :

$$\frac{H_b}{H_0} = (0.34 + 2.47\,m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88\,m} \quad \text{.....(19)}$$

m is the bottom slope, this research uses $m = 0$

Gourlay (1992) : $\frac{H_b}{H_0} = 0.478\left(\frac{H_0}{L_0}\right)^{-0.28}$ ....(20)

From these equations, the deep water wave length is calculated using the dispersion equations of linear wave theory, $L_0 = \frac{gT^2}{2\pi}$.

The breaker height calculation results using the empirical breaker height index equations are presented in Table (6).

Table (7) Breaker height from empirical equation.

H_0 (m)	T (sec)	$H_b(m)$			
		(17)	(18)	(19)	(20)
1	4.61	1.13	1.23	0.97	1.27
1.5	5.65	1.69	1.84	1.46	1.91
2	6.53	2.26	2.46	1.95	2.55
2.5	7.3	2.82	3.07	2.43	3.19
3	7.99	3.39	3.69	2.92	3.82

Note : (17), (18), (19), (20) are the equation code number.

Of the four breaker height index equations, (20) gives the largest and closest result to the breaker height in this study. Furthermore, the difference between the breaker height from this study and the breaker height from (20) was studied. The comparison results are presented in Table (8).

Table (8) Comparison between breaker height and Gourlay's breaker height

H_0 (m)	T (sec)	H_b (m)		δ (%)
		(13)	(20)	
1	4.61	1.37	1.27	7.24
1.5	5.65	2.05	1.91	7.24
2	6.53	2.73	2.55	7.24
2.5	7.3	3.42	3.19	7.24
3	7.99	4.1	3.82	7.24

Note : $\delta = \left|\frac{(13)-(20)}{(20)}\right| x100\ \%$

In Table (8), the breaker height of the results of this study is greater than the breaker height of (20) with a constant difference of 7.24%. To bring the results (13) closer to (20), it can be done by reducing the value of θ, when $\theta = 0.35$, it is obtained $\delta = 0.44$ %, but if $\frac{H_b}{h_b} = 10.7$, it shows that the (20) is for long wave.

XI. CONCLUSION

The first conclusion from this study is that the Kinematic Free Surface Boundary Condition has breaking characteristics and the breaker index equation can be formulated. Meanwhile, in the wave number conservation equation there is a breaker index in the form of a ratio between breaker depth and breaker length. The breaker height equation can be obtained by substituting the breaker length index in the energy conservation equation. With this procedure the breaker index equations for short waves are obtained.

From the results of the connectivity test on the resulting breaker index equations, it was found that there is a link between the breaker indexes. This also gives the conclusion that there is interaction between breaking parameters that the value of one breaking parameter depends on the value of another breaking parameter.

Thus it can also be concluded that breaking wave research, both laboratory and analytical research cannot be done separately, it should be done simultaneously for all breaker indexes or all breaking parameters.

Determining the deep water depth affects the breaking parameters, especially the breaker depth. Therefore it is estimated that the difference between the breaker depth index results of previous studies is due to differences in the determination of deep water depth.

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