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Based on Simulink Simulation of the Fuzzy PID Control for the TORA System

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***Keywords*— Translational oscillations with a rotational actuator (TORA) system, fuzzy PID (Proportion Integration Differentiation) control, triangle membership function, double S-type membership function, Simulink simulation**

***Abstract*— The purpose of this study is to control the TORA system through the MATLAB/Simulink simulation platform. The TORA system is a classical two-degree-of-freedom under-driven mechanical system, and this paper mainly focuses on the design of the control scheme for the swing angle of the ball. Firstly, the transfer function is established in the complex domain through the dynamical equations of the system. Then, the traditional PID controller, the triangular membership function fuzzy PID controller, and the double S type membership function fuzzy PID controller were designed. Finally, the simulation results are analyzed to compare the parameter performance of the three controllers. The fuzzy PID controller using the double S membership function is better than the other two controllers in terms of overshoot and adjustment time and has good practicability, stability, speed, and accuracy.*

I. INTRODUCTION

Translational oscillators with a rotational actuator (TORA) were originally used as a simplified model for studying the natural vibration phenomenon of dual spin spacecraft. Due to their outstanding characteristics such as light weight, low energy consumption, low cost, and strong flexibility, they have been widely used in aerospace [1, 2, 3], defense and military industry [4], mechanical manufacturing [5], and other fields. It is currently commonly used as a nonlinear benchmark system for nonlinear controller design, verification of the control performance of nonlinear control algorithms, or teaching research. This study simplifies the model of a dual-spin spacecraft to consist of an unpowered small car and a driven small ball [15].

The TORA system is a typical underactuated mechanical system that can serve as a benchmark system in control theory research for validating different algorithms. In the initial TORA system, the ball rotated in the horizontal plane, and currently, many scholars have studied the problem of the ball's rotation in the vertical plane. The control methods of the TORA system mainly include the PID method [6], feedback method [7], back-stepping method, energy-based control method [8], and sliding mode control method [9, 10, 11, 12].

This study first analyzes the dynamic equations of the vertically underactuated TORA system and then establishes a simulation model, mainly using traditional PID controllers and fuzzy PID controllers to control the TORA system. In order to further compare the performance of controllers, a comparison was made between triangular membership functions and double S-shaped membership functions in the design of fuzzy PID controllers [13, 14]. Finally, a simulation model was established using the Simulink module of MATLAB to conduct simulation experiments on the TORA system.

II. TORA SYSTEM MATHEMATICAL MODELING

The model parameters of the TORA system are: car mass M=1.3608 kg, ball mass m=0.096kg, counterclockwise rotation angle θ (rad) from the vertical downward direction, radius of ball rotation r=0.0592 m, distance of car movement x(m), input torque τ(N·m), car friction coefficient *f*=0.1N/m/sec, spring elasticity coefficient k=186.3N/m, center of mass rotational inertia J=0.0002175kg*m^2, and gravity acceleration g=9.8 N/ m^2. The TORA system is shown in Figure 1. It is composed of a small car in a horizontal direction and a small ball in a circular motion inside the car, and the left end of the car is connected to the wall through a spring. Drive the small ball in a circular motion to move the car horizontally. Dynamically model the TORA system using the Lagrange equation.

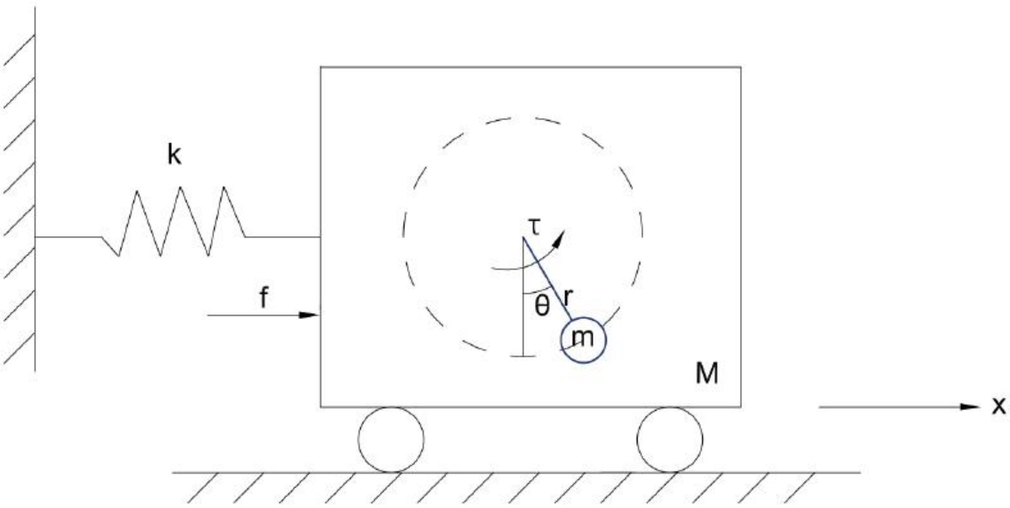

Fig.1 TORA System Schematic Diagram

Firstly, the sum of the kinetic energy of the car and the kinetic energy of the ball is the total kinetic energy K of the TORA system, that is:

$$\mathrm{K} = \frac{1}{2}(M+m)\dot{x}^2 + mr\dot{x}\dot{\theta}\theta + \frac{1}{2}(M+m)\dot{\theta}^2 + mgr\cos\theta - \frac{kx^2}{2} \quad (1)$$

We taking the horizontal plane as the reference plane for gravitational potential energy, the car moves on the horizontal plane and therefore do not possess gravitational potential energy. So the gravitational potential energy P of the TORA system is the sum of the gravitational potential energy of the ball and the elastic potential energy of the spring, that is:

$$\mathrm{P} = -mgr\cos\theta + \frac{kx^2}{2} \quad (2)$$

The Lagrangian operator function of the TORA system is

$$L = K - P \quad (3)$$

According to the Lagrangian equation, the dynamic equation of the TORA system is

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial \dot{x}} = f \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \dot{\theta}} = r \end{cases} \quad (4)$$

Substituting equations (1), (2), and (3) into equation (4) yields the dynamic model of the TORA system as follows

$$(M+m)\ddot{x} + mr\cos\theta\,\ddot{\theta} - mr\sin\theta\,\dot{\theta}^2 + kx = f \quad (5)$$

$$mr\cos\theta x + (mr^2 + J)\ddot{\theta} + mgr\,\ddot{s}\imath n\,\theta = \tau \quad (6)$$

In a vertical TORA system, our control objective is to place the system at a stable equilibrium point. When we substitute the following data ($\dot{x} = 0, \ddot{x} = 0, \dot{\theta} = 0, \ddot{\theta} = 0, \tau = 0$) into equation (1), we can obtain a stable equilibrium point$(\dot{x}, \ddot{x}, \dot{\theta}, \ddot{\theta}) = (0,0,0,0)$, that is, when the ball is in a vertical downward position.

At this point, we linearize the TORA system. When the small ball is infinitely close to the vertical downward position, i.e., θ is infinitely close to 0, then there is, $\cos\theta \approx 1, \sin\theta = \theta, (d\theta/dt)^2 = 0$ which is introduced into equations (5) and (6), resulting in

$$(M+m)\ddot{x} + mr\ddot{\theta} + kx = f \quad (7)$$

$$mr\ddot{x} + (mr^2 + J)\ddot{\theta} + mgr\theta = \tau \quad (8)$$

In the TORA system, when the car is moving horizontally, it will generate frictional force with the horizontal plane to hinder the car's movement. The frictional force in this article is

$$f = -\mu\dot{x} \quad (9)$$

Perform the Laplace transform on equations (7), (8), and (9) to obtain

$$(M+m)s^2X(s) + mrs^2\theta(s) + kX(s) = -\mu sX(s) \quad (10)$$

$$mrs^2X(s) + (mr^2 + J)s^2\theta(s) + mgr\theta(s) = \tau(s) \quad (11)$$

Equation (10) can be written in the following form:

$$\frac{x(s)}{\theta(s)} = -\frac{mrs^2}{(M+m)s^2 + \mu s + k} \quad (12)$$

Substituting equation (12) into equation (11) with the control input yields

$$\frac{\theta(s)}{\tau(s)} = \frac{\frac{(M+m)}{w}s^2 + \frac{\mu}{w} + \frac{k}{w}}{s^4 + \frac{\mu(mr^2+J)}{w}s^3 + \frac{[k(mr^2+J) + mgr(M+m)]}{w}s^2 + \mu\frac{mgr}{w}s + \frac{kmgr}{w}} \quad (13)$$

Among them, $w = [(M+m)(mr^2+J) - m^2r^2]$.

Substitute the parameters of the TORA system into equation (13) to obtain the open-loop transfer function for the swing angle θ and input torque τ of the ball, which is:

$$G(s) = \frac{\theta(s)}{\tau(s)} = \frac{2630s^2 + 180s + 336315}{s^4 + 0.07s^3 + 333s^2 + 10s + 18731} \quad (14)$$

III. TORA SYSTEM CONTROLLER PRINCIPLE

3.1 Principle of Traditional PID Control

In general control systems, the most common control method is to use traditional PID control (Figure 2).

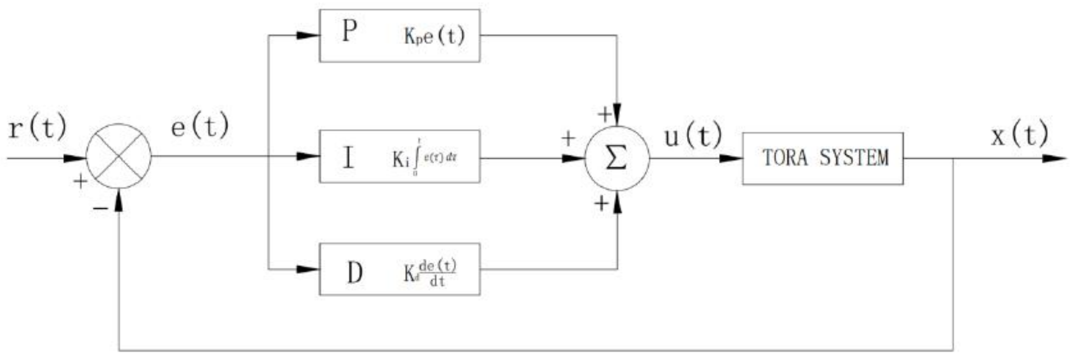

Fig.2 Principle Framework Diagram of Traditional PID Control System

The traditional PID controller is a linear controller that forms the control deviation e of the controller based on the difference between the given input value r(t) and the actual output value u(t). The output error equation of the traditional PID controller is as follows:

$$e(t) = r(t) - u(t) \quad (15)$$

The control equation of a traditional PID controller is as follows:

$$u(t) = K_p e(t) + K_I \int_0^t e(t)dt + \frac{K_D de(t)}{dt} \quad (16)$$

In the above equation, K_p is a proportional coefficient, K_I is an integral coefficient, and K_D is a differential coefficient. The proportional coefficient is the amplification factor of the difference between the preset value and the feedback value, and the larger the proportion, the higher the adjustment sensitivity. The integration coefficient accumulates the difference between the preset value and the feedback value over time, but there is a significant lag. The differential coefficient is the rate of change of the research object (i.e., the difference between the two differences before and after), and a corresponding adjustment action is given in advance based on the rate of change of the difference. According to the control laws of each coefficient, it can be seen that the proportional coefficient makes the reaction faster, the differential coefficient makes the reaction earlier, and the integral coefficient makes the reaction lag. Within a certain range, the larger the value of K_p and K_D, the better the adjustment effect.

3.2 Principle of Fuzzy PID Control

Fuzzy PID control is a control method that combines fuzzy logic and classical PID controllers. A PID controller is a common feedback controller that adjusts the control output based on the size of the error signal to make the system output value close to the expected value. The PID controller consists of a proportional term (P), an integral term (I), and a differential term (D). Fuzzy control mainly consists of three steps: fuzzification, determining fuzzy rules, and deblurring. The control framework diagram for fuzzy PID control is shown as Figure 3.

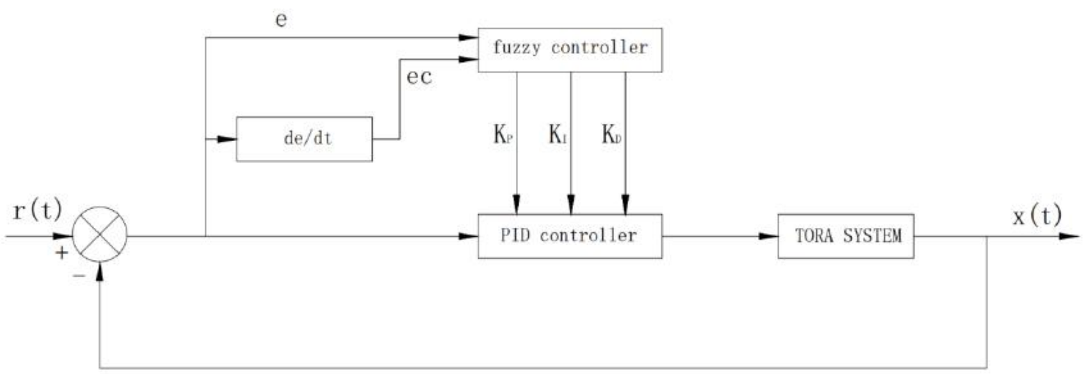

Fig.3 Fuzzy PID Control Block Diagram

In the above figure, e represents the control error, and ec represents the rate of change of the control error. In a fuzzy PID control system, e and ec are used as the two input variables of the controller, and the differential coefficients (K_p 、K_I 、K_D) are used as the three output variables of the controller.

IV. DESIGN OF FUZZY PID CONTROLLER

In this article, the fuzzy PID controller is a two-input, three-output type. The basic domain of error e is taken as [-0.6, 0.6], the basic domain of error rate of change is taken as [-0.6, 0.6], and the domain of output variables is taken as [-3, 3]. Divide the fuzzy domain of input and output variables into 7 fuzzy subsets, namely NB, NM, NS, ZO, PS, PM, and PB.

In fuzzy set theory, the membership function refers to a function used to measure the degree of membership of an element to a fuzzy set. It is one of the most basic concepts in fuzzy sets, used to describe the degree of membership of elements in a certain fuzzy set. In this study, the triangular membership function (trimf) and double S-type membership function (dsigmf) are used as the membership functions of the fuzzy PID controller. Compare the results of two types of membership functions and choose the one with the best control effect as the membership function of the fuzzy PID controller. Their membership function diagrams are as follows (Figure 4, 5):

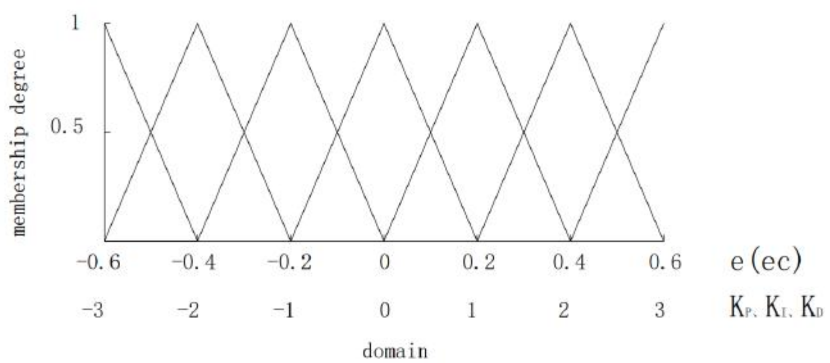

Fig.4 Triangle Membership Function Diagram

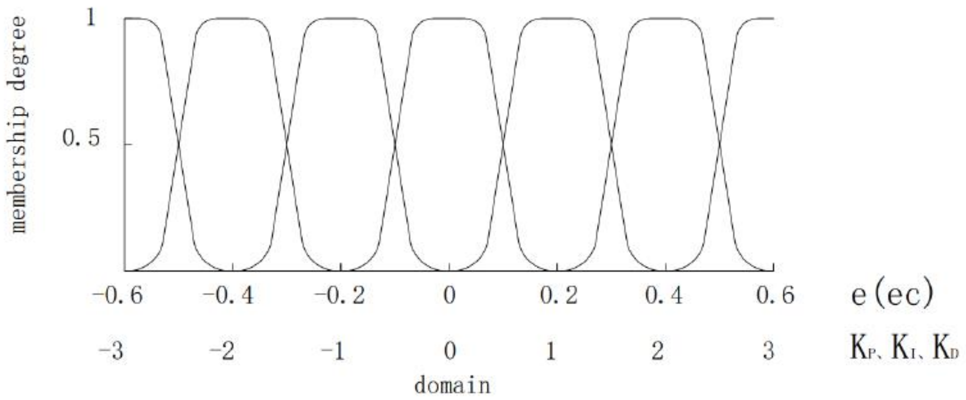

Fig.5 Double S-shaped Membership Function Diagram

Control rules are usually established by experts based on long-term experience. By determining the input and output quantities of the fuzzy controller, a fuzzy subset is determined. Based on the fuzzy rules established above, a rule table for the fuzzy PID controller is established, as shown below (Table 1, 2, 3).

Table 1 K_p Fuzzy Rule Table

K_P E / EC	NB	NM	NS	O	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	O
NM	PB	PB	PB	PB	PM	O	O
NS	PM	PM	PM	PM	O	PS	PS
O	PM	PM	PS	O	NS	NS	NM
PS	PS	PS	O	NS	NM	NM	NM
PM	PS	O	NS	NM	NM	NM	NB
PB	O	O	NM	NM	NM	NB	NB

Table 2 K_I Fuzzy Rule Table

K_I E / EC	NB	NM	NS	O	PS	PM	PB
NB	NB	NB	NM	NM	NS	O	O
NM	NB	NB	NM	NS	NS	O	O
NS	NB	NM	NS	NS	O	PS	PS
O	NM	NM	NS	O	PS	PM	PM
PS	NM	NS	O	PS	PS	PM	PB
PM	O	O	PS	NM	PM	PB	PB
PB	O	O	PS	PM	PM	PB	PB

Table 3 K_D Fuzzy Rule Table

K_D E / EC	NB	NM	NS	O	PS	PM	PB
NB	PS	NS	NB	NB	NB	NM	PS
NM	PS	NS	NB	NM	NM	NS	O
NS	O	NS	NM	NM	NM	NS	O
O	O	NS	NS	NS	NS	NS	O
PS	O	O	O	O	O	O	O
PM	PB	PS	PS	PS	PS	PS	PB
PB	PB	PM	PM	PM	PS	PS	PB

This design system uses the Mamdani inference method to perform fuzzy inference on the established fuzzy rules in order to obtain control variables.

V. TORA SYSTEM SIMULATION AND RESULT ANALYSIS

This study based on the established control model and controller, conduct simulation analysis on the TORA system. The control model was built using the Simulink module of MATLAB, and two different membership functions of fuzzy PID and traditional PID were compared and analyzed. The system control model built is shown in Figure 6,7,8.

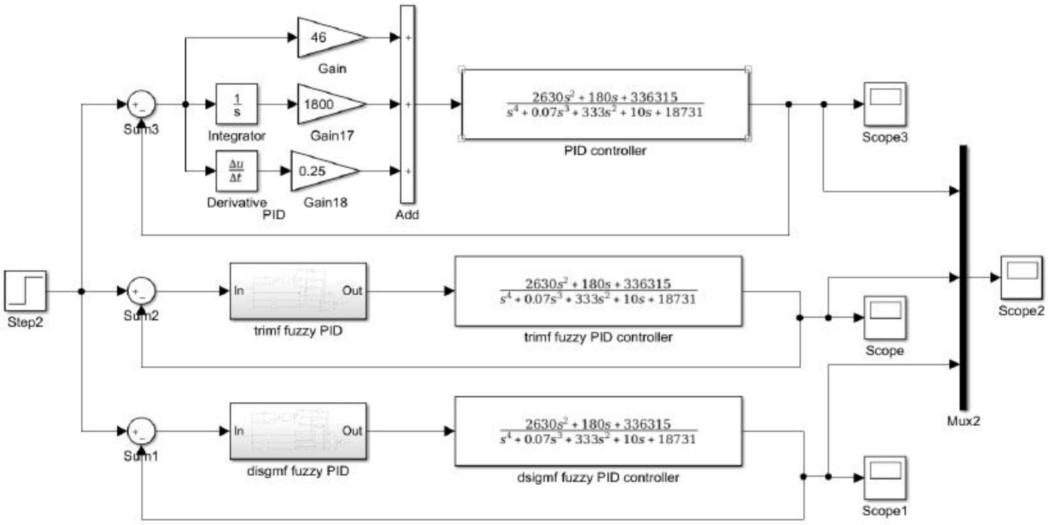

Fig.6 Simulink Simulations of Traditional PID and Fuzzy PID Control Systems (Ball Angle)

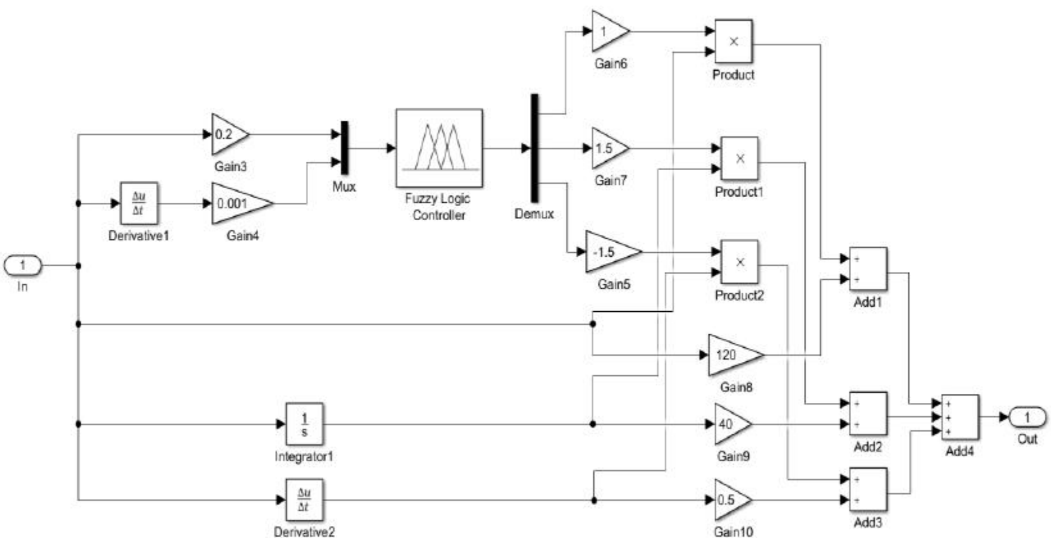

Fig.7 Simulink Simulation of Fuzzy PID Controller for Ball Angle

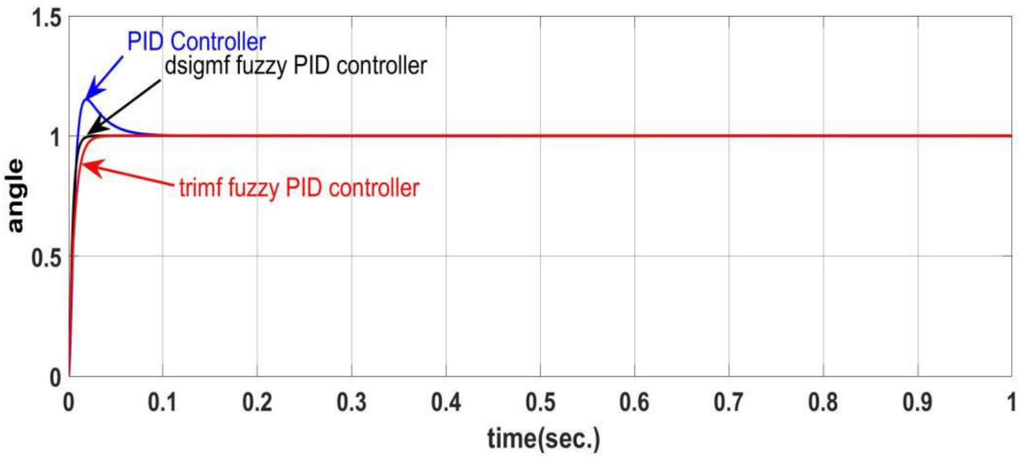

Fig.8 Response Curves of Traditional PID and Fuzzy PID Controller Systems (Ball Angle)

Through simulation using MATLAB/Simulink modules, it has been found that with appropriate parameters, traditional PID controllers have good control effects on control objects that can assume precise mathematical models. They can meet the requirements of system control accuracy and rise time, but there are problems such as long adjustment times. As shown in Table 4, two fuzzy PID controllers with different membership functions can adjust the PID parameters of the system in real time. By doing so, the PID parameters can be more suitable for the control requirements of the system, resulting in better control effects than traditional PID controllers. Moreover, the fuzzy PID controller using dual S-type membership functions performs the best among the three controllers.

Table 4 Comparison of Performance Parameters

Method	Rise time (s)	Overshoot (%)	Adjusting time (s)
Tradition PID	0.02	17	0.09
Fuzzy PID(trimf)	0.08	1	0.08
Fuzzy PID(dsigmf)	0.06	0.5	0.06

VI. CONCLUSION

The inherent TORA system is a typical two-degree-of-freedom underactuated mechanical system. This study analyzes the motion of the TORA system, establishes a mathematical model of the TORA system using the Lagrange method, and designs a PID controller and two types of fuzzy PID controllers to control the TORA system. By adjusting their proportional constant, integral constant, and differential constant parameters, fuzzifying the fuzzy PID controller, designing fuzzy rules, and defuzzifying, them the TORA system can quickly and accurately reach a stable state, ultimately achieving a stable system.

The superiority of the fuzzy PID controller with dual S-type membership functions over the other two controllers was verified through Simulink simulation. Under the action of the fuzzy PID controller with dual S-type membership functions, the system has fast response speed, short adjustment time, and high steady-state accuracy, achieving the expected goals.

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